Joint Array and Spatial Sparsity Based Optimisation for DoA Estimation

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Abstract

Traditional spatial sparse techniques for DoA (Direction of Arrival) estimation are implemented by full arrays. However in practice, it is desirable to use as few sensors as possible to reduce the cost for manufacturing the array or to counter against sensor failure. As a result, joint optimisation of sparse array and spatial sparsity becomes an ideal alternative. In most of existing methods, these two kinds of sparsity are studied separately. This paper proposes a joint approach which achieves source detection in a subset of space using partial array sensors. The core idea is to use the weight coefficients obtained in sparse array optimisation to scale the model for the sparse reconstruction based DoA estimation. Compressive sensing based optimisation is used for both steps. The numerical results of DoA estimation for both stationary source and moving source are used to demonstrate the feasibility of this joint model.

Joint Method

- **Signal model**

$$y_k = Ax_k + n_k$$

$y_k : (y_{k_1}, y_{k_2}, \cdots, y_{k_N})$ at each time step k where N is the number of sensors.

$x_k : (x_{k_1}, x_{k_2}, \cdots, x_{k_N})$ where M is the potential source directions.

A contains steering vectors from -90 to +90 degrees with size of $N \times M$

- **Compressive sensing based sparse array**

Minimisation of the l_1 norm of vector weight coefficients, $w = w_R + w_I i$

$$\min \ \|w_R\|_1 + \|w_I\|_1$$
$$\text{subject to } \ \|f - (w_R + w_I i)^H A\|_2 \le \alpha \quad (1)$$

f is the vector holding the desired beam response.

- **Updated observed signal**

$$W = \begin{pmatrix} |w_1| & & \\ & \cdots & \\ & & |w_N| \end{pmatrix}, \quad y_{new} = W y_k$$

$(w_1, w_2, \cdots, w_N)$ is the result obtained by sparse array optimisation.

- **Joint optimisation approach**

The least absolute shrinkage and selection operator (LASSO) function promoting spatial sparsity is defined by

$$\min \ \|y_{new} - Ax_k\|_2^2 + \mu \|x_k\|_1 \quad (2)$$

where μ is to control sparsity. Both cost functions (1) and (2) are optimised by the CVX toolbox in Matlab.

1. Set $k=1$ and create an initial beam response as input f of (1), corresponding to a randomly selected direction.
2. Implement two-step optimisation with the updated observed signal and obtain the possible angles x_k by solving (2).
3. Replace f by $(Ax_k)^H A$, and set $k = k+1$.
4. Repeat stages 2 to 3 until k reaches the predefined maximum time step K.

Numerical Results

- **Experiment setup**

Implement DoA estimations of stationary source and moving source. The underwater speed of sound used in this model is assumed to be 1500 m/s, and the frequency of the sources is 200 Hz. In the noisy case, the level of noise in terms of Signal to Noise Ratio (SNR) is 20 dB.

A grid of 100 potential sensors are used. For stationary source estimation, the inter-senor spacing is 0.1 λ (λ is the wavelength) and maximum running step is $K=21$. For moving source estimation, the inter-senor spacing is 0.05 λ and maximum running step is $K=100$.

In order to measure the accuracy of DoA estimation, we define the Mean Square Error (MSE) and Standard Error (SE) as follows,

$$MSE = 20\log_{10}\left(\frac{\| f - w^H A \|_2^2}{M}\right)dB, \quad SE = \sqrt{\left(\frac{\| y_k^H A - (Ax_x)^H A \|_2^2}{M}\right)}\text{degree}$$

- **Results**

Source	MSE(dB)	SE(degree)	Active sensors	SNR(dB)
Stationary	-66.7	0.49	36/100	-
Stationary	-70.7	0.55	37/100	20
Moving	-70.6	0.27	22/100	-
Moving	-71.4	0.31	22/100	20

Table 1. Simulation results for stationary source and moving source

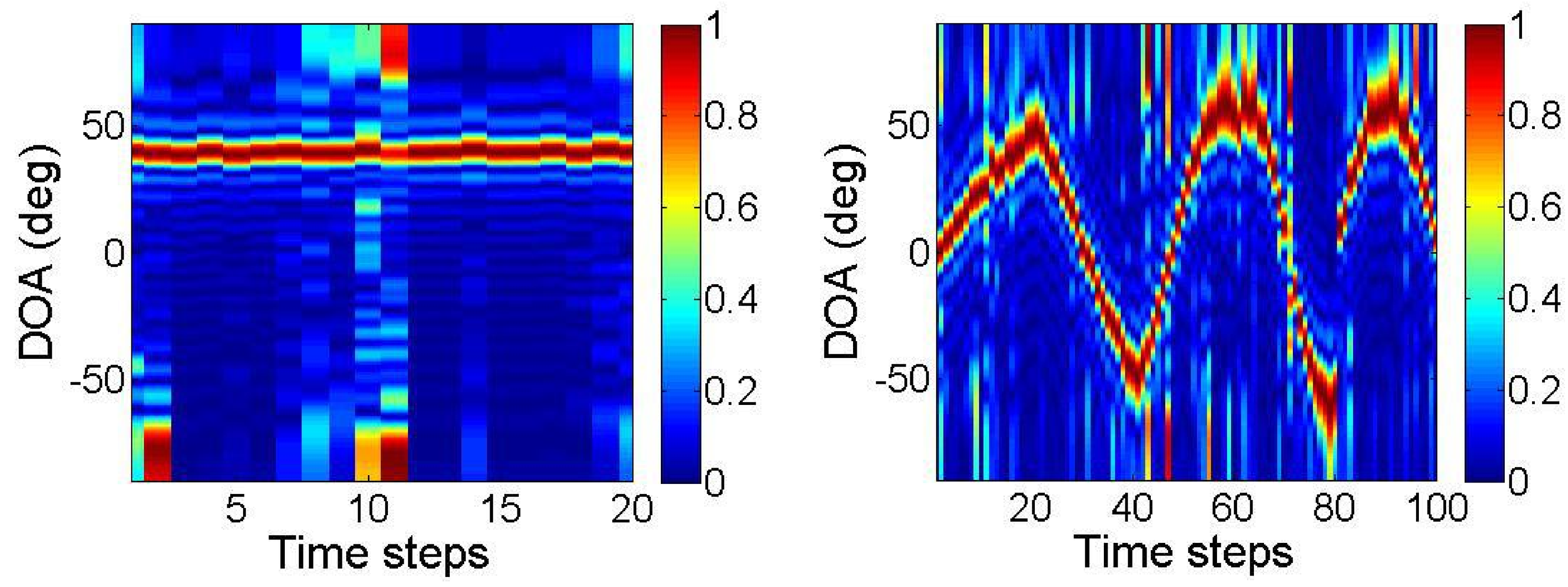

Figure 1. DoA estimations for the sources with noise

Conclusions

A new method has been presented to exploit the joint sparsity in array design and source localisation. The method is operated in a two-step iterative process, of which the first step is to find the sensors to be used from the array and the second step is to perform source localisation with the LASSO algorithm with the selected sensors. The two steps are iterated in an alternating manner. The algorithm can start with a random guess of the DoA of the source when performing the optimisation for the sparse array, but eventually find the DoA of the source with the sparse reconstruction algorithm. The results evaluated for both stationary source and moving source show good performance of the proposed method.

Reference

1. E. Zochmann, P. Gerstoft, and C. F. Mecklenbräuker, "Density evolution of sparse source signals," *in 2015 3rd International Workshop on Compressed Sensing Theory and its Applications to Radar, Sonar and Remote Sensing (CoSeRa). IEEE, 2015*, pp. 124–128.

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