Knowledge-aided Adaptive Detection with Multipath Exploitation Radar

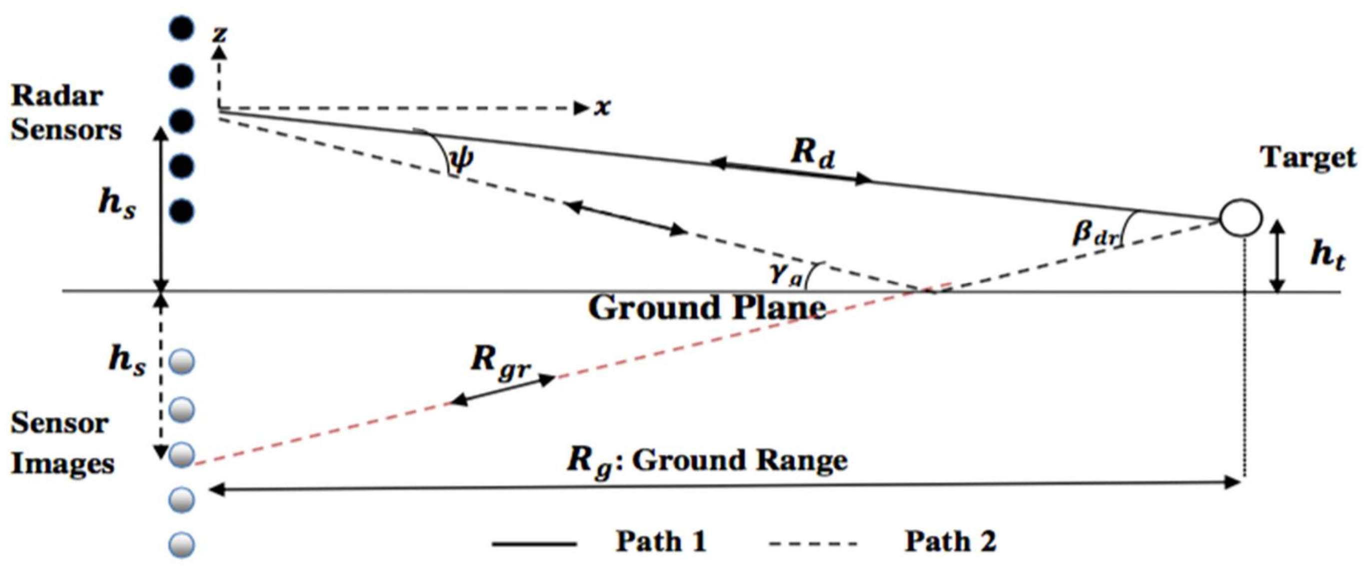
Utku Kumbul, Harun Taha Hayvaci

TOBB University of Economics and Technology, Ankara, Turkey

Abstract

The adaptive detection of point-like targets in the presence of multipath is considered in this article. Target return signal is modeled as the sum of direct path return and reflected path return signals under the assumption of a zero-mean complex circular Gaussian noise with an unknown covariance matrix. A new approach to exploit multipath returns in target detection with an adaptive regime is studied. The novelty of this approach is that the multipath returns are exploited with a priori knowledge of the reflecting environment, so that we have the knowledge of the reflected steering vector for a known actual direct path steering vector. As a case study, we analyze a radar-target scenario over a flat conducting surface

Formulation Of The Detection Problem



$$H_0:\ \begin{cases} \boldsymbol{r} = \boldsymbol{n} \\ \boldsymbol{r}_k = \boldsymbol{n}_k, \quad k = 1,\dots,K \end{cases}$$

$$H_1:\ \begin{cases} \boldsymbol{r} = \alpha_1\boldsymbol{p} + \alpha_2\boldsymbol{s} + \boldsymbol{n} \\ \boldsymbol{r}_k = \boldsymbol{n}_k, \quad k = 1,\dots,K \end{cases}$$

where

- $\boldsymbol{p} \in \mathbb{C}^{N\times1}$, $\|\boldsymbol{p}\|^2 = 1$, is the target steering vector associated with the direct path return.
- $\boldsymbol{s} \in \mathbb{C}^{N\times1}$, $\|\boldsymbol{s}\|^2 = 1$, is the reflected path target steering vector.
- α_1 and $\alpha_2 \in \mathbb{C}^{N\times1}$ are the unknown deterministic parameters for attenuation in propagation associated with the direct path and reflected path, respectively;
- $\boldsymbol{n}$ and $\boldsymbol{n}_k \in \mathbb{C}^{N\times1}$, $k = 1,\dots,K$, denote the noise contributions which are assumed to be independent and identically distributed (iid), complex normal random vectors with zero-mean and covariance matrix $\boldsymbol{C} \succ \boldsymbol{0}$. Thus, primary data and secondary data set are assumed to have the same covariance matrix, i.e. $E[\boldsymbol{n}\boldsymbol{n}^\dagger] = \mathbb{E}[\boldsymbol{n}_k\boldsymbol{n}_k^\dagger] = \boldsymbol{C}$.

Detector Design

Based on the hypothesis test, we design a new adaptive detector as

$$\frac{\max\limits_{\alpha_1,\alpha_2\in\mathbb{C},\boldsymbol{C}} p_1(\boldsymbol{r};\alpha_1,\alpha_2,\boldsymbol{C})}{\max\limits_{\boldsymbol{C}} p_0(\boldsymbol{r};\boldsymbol{C})} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_1$$

Where

$$p_0(\boldsymbol{r};\boldsymbol{C}) = \frac{1}{\det(\pi\boldsymbol{C})}\exp\left\{-\boldsymbol{r}^\dagger\boldsymbol{C}^{-1}\boldsymbol{r}\right\}$$

$$p_1(\boldsymbol{r};\alpha_1,\alpha_2,\boldsymbol{C}) = \frac{1}{\det(\pi\boldsymbol{C})}\exp\left\{-\bar{\boldsymbol{r}}^\dagger\boldsymbol{C}^{-1}\bar{\boldsymbol{r}}\right\} \quad \bar{\boldsymbol{r}} = \boldsymbol{r} - \alpha_1\boldsymbol{p} - \alpha_2\boldsymbol{s}$$

To maximize likelihood function p_1, the resulting maximum likelihood estimates of α_1 and α_2 are given as

$$\hat{\alpha}_1 = \frac{p^\dagger C^{-1}r - \frac{p^\dagger C^{-1}s}{s^\dagger C^{-1}s}s^\dagger C^{-1}r}{p^\dagger C^{-1}p - \frac{p^\dagger C^{-1}s}{s^\dagger C^{-1}s}s^\dagger C^{-1}p} \qquad \hat{\alpha}_2 = \frac{s^\dagger C^{-1}r - p^\dagger C^{-1}r\frac{s^\dagger C^{-1}p}{p^\dagger C^{-1}p}}{s^\dagger C^{-1}s - p^\dagger C^{-1}s\frac{s^\dagger C^{-1}p}{p^\dagger C^{-1}p}}$$

Inserting estimated covariance matrix **S** and maximum likelihood estimates of α_1 and α_2 into the log-likelihood ratio, we have our new adaptive detector as

$$\frac{\frac{|\boldsymbol{p}^\dagger\boldsymbol{S}^{-1}\boldsymbol{r}|^2}{\boldsymbol{p}^\dagger\boldsymbol{S}^{-1}\boldsymbol{p}} + \frac{|\boldsymbol{s}^\dagger\boldsymbol{S}^{-1}\boldsymbol{r}|^2}{\boldsymbol{s}^\dagger\boldsymbol{S}^{-1}\boldsymbol{s}} - 2\Re\left\{\frac{(\boldsymbol{p}^\dagger\boldsymbol{S}^{-1}\boldsymbol{r})(\boldsymbol{r}^\dagger\boldsymbol{S}^{-1}\boldsymbol{s})}{\boldsymbol{p}^\dagger\boldsymbol{S}^{-1}\boldsymbol{s}}\right\}\cos^2\theta}{1-\cos^2\theta}$$

Where θ is the angle between direct-path and reflected-path steering vectors in the whitened observation space, so that we have

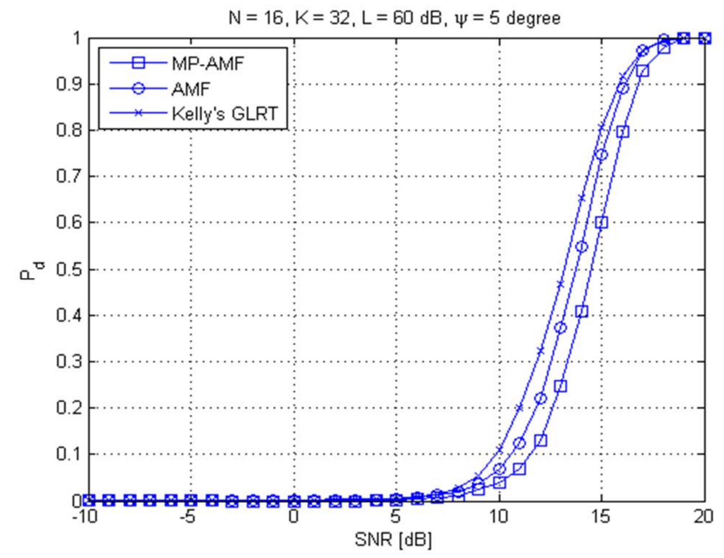
$$\cos^2\theta = \frac{\left|\boldsymbol{s}^\dagger\boldsymbol{S}^{-1}\boldsymbol{p}\right|^2}{\left(\boldsymbol{p}^\dagger\boldsymbol{S}^{-1}\boldsymbol{p}\right)\left(\boldsymbol{s}^\dagger\boldsymbol{S}^{-1}\boldsymbol{s}\right)}$$

And **S** is the estimated covariance matrix and computed as the sample covariance matrix of the secondary data

$$\boldsymbol{S} = \frac{1}{K}\sum_{k=1}^{K}\boldsymbol{r}_k\boldsymbol{r}_k^\dagger$$

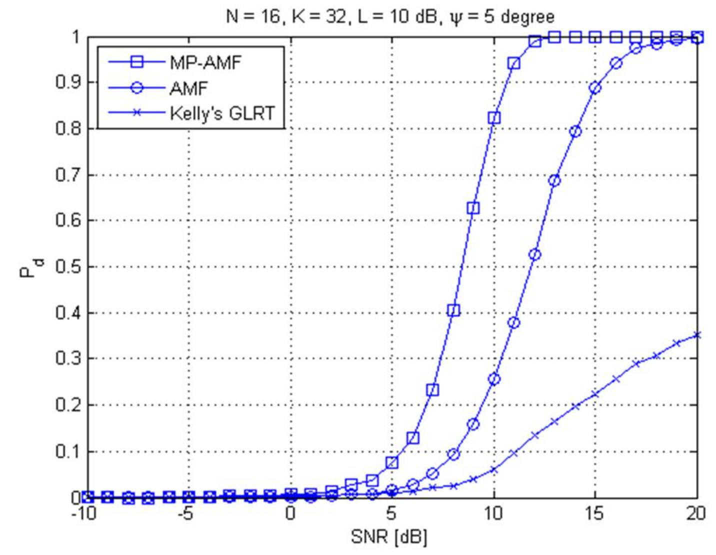
Performance

We assess the performance of proposed detector, which we call Multipath Adaptive Matched Filter (MP-AMF), by comparing Kelly's detector and the AMF in terms of probability of detection

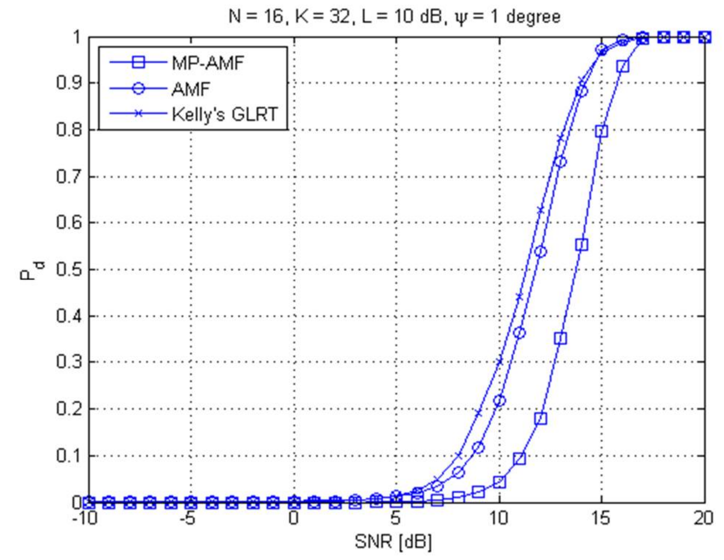


When there is no multipath, i.e. L=60 dB

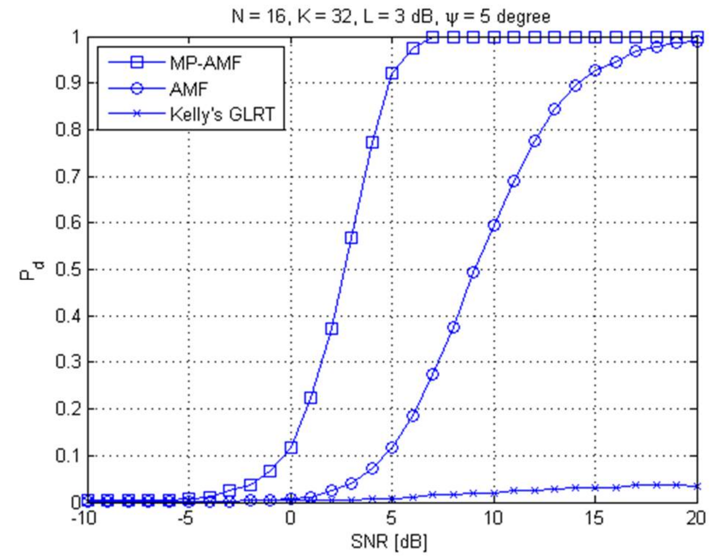
where $|\alpha_2| = |\alpha_1|/\sqrt{L}$



When multipath return has sufficient power to be exploited, i.e. L=10 dB



When multipath returns are not distinguishable as difference of angles of arrival between direct path and reflected path set to ψ=1°



When the reflected path has 3 dB loss relative to direct path

Conclusion

We have introduced a new adaptive detector, MP-AMF. It has been shown that when there are significant distinguishable multipath returns, the MP-AMF detector outperforms the AMF and Kelly's detector with a significant SNR gain at higher P_d rates. However, at highly clumped multipath structures and very low power reflected path returns, MP-AMF has slight performance degradation with respect to the AMF and Kelly's detector. This is expected since the MP-AMF estimates a second unknown parameter, i.e. reflected path signal strength, which requires a certain level of SNR and partially distinguishable multipath structure