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A Cognitive Stepped Frequency Strategy for HRRP Estimation

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Introduction

- Classic approaches adopted to acquire **HRRPs** employ radar pulses with large instantaneous bandwidths.
- This is a limiting factor due to the **high cost** of both transmitters and receivers.
- To overcome this drawback the **SF** technology has been widely used to perform HRRPs recovery.

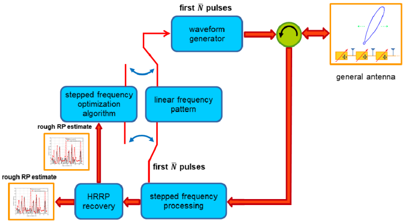
  It exploits interpulse modulations to synthesize a very large radar bandwidth involving narrowband pulses. The interpulse modulation rules the carrier frequency of each transmitted pulse allowing, at the receiver side, the use of a tunable narrow bandwidth detector matched to a specific carrier frequency. Then, the received signals collected from the different pulses are ***coherently combined*** *to achieve a fine range resolution.*

- In the conventional SF approach, the carrier frequency of successive pulses is increased of a constant offset, emulating a wideband chirp signal.

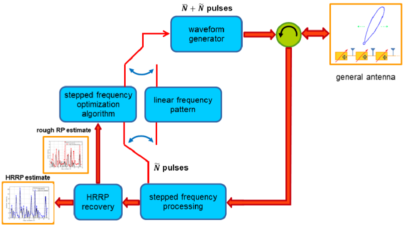
The frequency hopping could be arbitrary and represents a degree of freedom

Introduction

- The idea is to **dynamically optimize the probing waveform** exploiting feedback information about the target RP aimed at enhancing target RP estimation accuracy.
- It is assumed that a **RP target prediction** is available. Then, the SF hopping pattern is designed to **minimize the predicted CRLB** associated with the target RP.
- The transmission process is composed of two phases:
  1. a linear SF pattern is transmitted in order to acquire a reliable RP prediction;
  2. the optimized SF pattern waveform is created.



(a) perception cycle



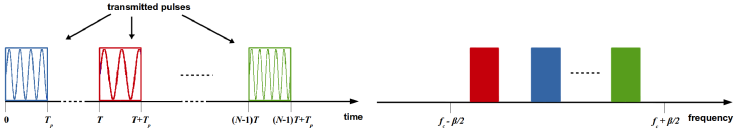
(b) action cycle

SF Signal Model

A radar which transmits a coherent burst of N **frequency modulated pulses** of duration T_p and PRI T is considered.

The **carrier frequency of the n-th pulse** is $f_n \in [f_c - \beta/2, f_c + \beta/2]$, $n = 1, \ldots, N$, with f_c the central carrier frequency.

The transmitted frequencies belong to **a discrete set,** $f_n = f_c + c_n\beta/2$ with $c_n \in \mathcal{F} = \{-1, -(N-1)/N, \ldots, 0, \ldots, (N-1)/N, 1\}$.



The n-th transmitted pulse is

$$s(n,t) = \frac{\sqrt{P}}{\sqrt{T_p}} \text{rect}\left(\frac{t - nT - T_p/2}{T_p}\right) e^{j2\pi(f_c + c_n\beta)(t - nT)}$$

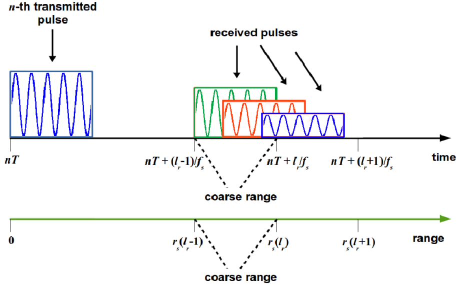
where:

- P is the radar transmit peak power;
- $\text{rect}(t) = 1$, $-0.5 \leq t \leq 0.5$, and 0 otherwise.

SF Signal Model

The waveform at the receiver end is down-converted to baseband, undergoes a filtering operation, and then it is sampled at the fast-time instants $t_s(n, l_r)$ where l_r indexes the **coarse range cells.**

The echoes from the scatterers whose range lies within $[r_s(l_r - 1), r_s(l_r)]$, i.e. belonging to the l_r-th coarse range bin, contribute to the measurement at the same time instant $t_s(n, l_r)$.



Goal: to extract the HRRP of the scatterers contained in a coarse range cell, l_r, suitably processing the corresponding data, $y(n, l_r)$, $n = 1, \ldots, N$.

SF Signal Model

The **discrete-time echo signal** from the k-th stationary scatterer located at the range r_k corresponding to a coarse range cell l_r can be expressed as

$$y_k(n, l_r) = \gamma_k \sqrt{P} e^{-j\frac{4\pi}{c}(f_c + c_n \beta) R_k}, \quad n = 1, \dots, N, \ k = 1, \dots, K,$$

where:

- R_k: high-resolution range displacement of the k-th scatterer from the coarse range bin;
- γ_k: scattering coefficient of the k-th scatterer from the coarse range bin;

Hence, in the presence of K **scatterers** within the course range of interest the useful received signal can be written as

$$y(n) = \sqrt{P} \sum_{k=1}^{K} \bar{\gamma}_k e^{j\zeta_k c_n}, \quad n = 1, \dots, N$$

SF Signal Model

The noisy version of the received signal can be expressed in a **vectorial form** as

$$\boldsymbol{y} = \sqrt{P}\sum_{k=1}^{K}\boldsymbol{h}(R_k, \boldsymbol{c})\bar{\gamma}_k + \boldsymbol{n} = \boldsymbol{H}(\boldsymbol{r}, \boldsymbol{c})\bar{\boldsymbol{\gamma}} + \boldsymbol{n}$$

with

- $\boldsymbol{r} = [R_1, \ldots, R_K]^T \in [0, c\,T_p/2]^K$: high-resolution range displacements;
- $\bar{\boldsymbol{\gamma}} = [\bar{\gamma}_1, \ldots, \bar{\gamma}_K]^T \in \mathbb{C}^K$: reflectivities of the scatterers within the coarse range bin under test;
- $\boldsymbol{c} = [c_1, \ldots, c_N]^T \in \mathcal{F}^N$: frequency modulation codes,
- $\boldsymbol{n} \in \mathbb{C}^N$: received disturbance signal, modeled as a zero mean, white, complex circularly symmetric Gaussian vector;
- $\boldsymbol{h}(R, \boldsymbol{c}) = \left[e^{-j4\pi\frac{\beta}{c}R\,c_1}, \ldots, e^{-j4\pi\frac{\beta}{c}R\,c_N}\right]^T \in \mathbb{C}^N$: normalized response (target steering vector) of a scatterer located at range increment R;
- $\boldsymbol{H}(\boldsymbol{r}, \boldsymbol{c}) = \sqrt{P}\,[\boldsymbol{h}(R_1, \boldsymbol{c}), \boldsymbol{h}(R_2, \boldsymbol{c}), \ldots, \boldsymbol{h}(R_K, \boldsymbol{c})] \in \mathbb{C}^{N\times K}$: model matrix.

HRRP Recovery Strategy

Several algorithms have been proposed in the open literature to estimate the unknown model parameters:

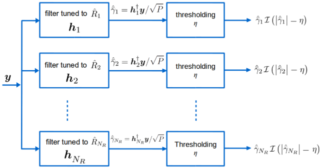
1. filter bank;
2. Iterative Adaptive Approach (IAA).
3. BIC-based IAA.

[1] W. Roberts, P. Stoica, J. Li, T. Yardibi, and F.A. Sadjadi, "Iterative Adaptive Approaches to MIMO Radar Imaging", *IEEE Journal of Selected Topics in Signal Processing*, Vol. 4, No. 1, pp. 5-20, 2010.

Bank Filter Processing

This strategy **exploits a collection of filters** $\{\boldsymbol{h}_1, \ldots, \boldsymbol{h}_{N_R}\}$, each tuned to a specific range increment of the grid, to process the data and estimate the reflectivity of the scatterer located at $R = \hat{R}_i$ as

$$\hat{\hat{\gamma}}_i = \boldsymbol{h}_i^\dagger \boldsymbol{y}/\sqrt{P}, \quad i = 1, \ldots, N_R.$$



IAA Processing

The IAA tries to enhance HRRP recovery, **iteratively refining the estimates** obtained through the matched filters tuned to the specific range increments.

Focusing on the i-th scatterer return, the idea is to **consider the echoes from the other possible targets as interference,** assuming that the phases of $\bar{\gamma}_l$, $l \neq i$, are i.i.d. random variables uniformly distributed over $[0, 2\pi[$.

The estimator of the scattering coefficient $\bar{\gamma}_i$ is given by

$$\hat{\bar{\gamma}}_i = \frac{1}{\sqrt{P}} \frac{\boldsymbol{h}_i^\dagger \boldsymbol{R}^{-1} \boldsymbol{y}}{\boldsymbol{h}_i^\dagger \boldsymbol{R}^{-1} \boldsymbol{h}_i}, \quad i = 1, \ldots, N_R$$

where

$$\boldsymbol{R} = P \sum_{i=1}^{N_R} |\bar{\gamma}_i|^2 \, \boldsymbol{h}_i \boldsymbol{h}_i^\dagger + \sigma^2 \boldsymbol{I}$$

Since $\boldsymbol{R}$ depends on the unknowns $|\bar{\gamma}_i|$, $i = 1, \ldots, N_R$, the IAA **iteratively replaces** the scattering coefficient estimates $\bar{\gamma}_i$ with $\hat{\bar{\gamma}}_i$ initializing the process with the filter bank estimates.

BIC-based IAA Processing

The recovery process can be enhanced accounting for the **sparsity information in the RP.**

A **sequential procedure** jointly exploiting the HRRP estimate provided by the IAA strategy and the Bayesian Information Criterion (BIC) framework is adopted at the estimation stage:

1. The coarse range cell of interest is divided in N_c disjoint range cells;
2. Each cell (containing at most one scatterer by assumption) is discretized with M_s sub-bins;
3. For each possible candidate scatterer, the objective involved BIC is computed;
4. The scatterer with sub-range index that minimizes the objective involved BIC is selected and removed for the possible candidates.

Frequency Hopping Optimization

The idea followed to design the optimized frequency hopping patterns is to choose the **codes $\boldsymbol{c}$ that minimize the CRLB of the unknown parameters**, i.e. $\boldsymbol{r}$ and $\bar{\gamma}$.

The CRLB yields a **lower bound to the covariance matrix of any unbiased estimator**.

The lower bound to the MSE can be considered as figure of merit:

$$\mathrm{tr}\left\{\boldsymbol{J}^{-1}\left(\boldsymbol{c},\boldsymbol{r},\bar{\gamma}\right)\right\}$$

- $\boldsymbol{J}$: FIM associated with the unknown parameters.

The frequency hopping pattern **influences the HRRP estimation accuracy.**

Frequency Hopping Optimization

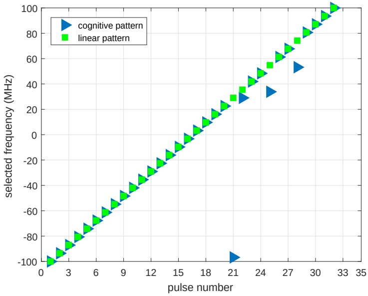
Since, the bound to the MSE **depends on the unknown parameters** $\boldsymbol{c}$, $\boldsymbol{r}$, and $\bar{\gamma}$, it is not usable in its current form.

To overcome this shortcoming, it is assumed that **a rough prediction of the target profile is available** at the transmitter side.

Hence the predicted CRLB is optimized.

More in details, the imaging of the target under illumination, is performed transmitting **N pulses:**

- The first $\bar{N}$ frequencies follow a classic linear stepped pattern to obtain a rough initial estimate of the RP.
- The frequencies of the other $\tilde{N}$ pulses are chosen to minimize the predicted CRLB.



Frequency Hopping Optimization

The problem to handle is **NP-hard**, therefore:

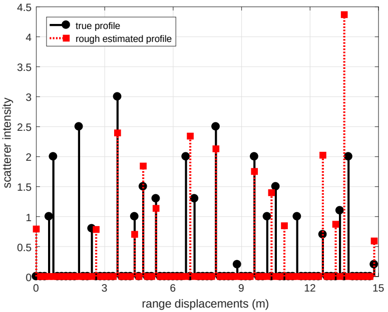
- An **efficient and effective iterative procedure** based on the Coordinate Descent paradigm is introduced;
- Each iteration of this procedure requires the solution of $\tilde{N}$ **univariate problems** in a sequence;
- Each problem addresses the optimization of the objective over the frequency of a specific pulse **keeping fixed the remaining** frequency pattern.

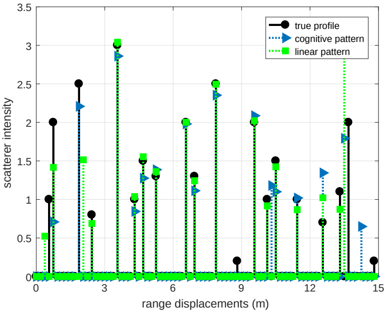
Performance Analysis

Radar system parameters:

- $N = 32$ (with $\bar{N} = 20$ and $\tilde{N} = 12$);
- $\beta = 200$ MHz;
- $f_c = 9$ GHz;
- $T_p = 0.1$ μs;
- $T = 1$ μs.

The reference RP is composed of 20 scatterers comprising $M_s = 4$ sub-bins, whose phases are modeled as i.i.d. uniform random variables in $[0, 2\pi]$.





Performance Analysis

To evaluate the performance of the proposed cognitive algorithm, the **Root Mean Square Error (RMSE)** of the RP estimate is considered as figure of merit:

$$\text{RMSE} = \sqrt{\mathbb{E}\left[\|\bar{\boldsymbol{\gamma}}^R - \hat{\bar{\boldsymbol{\gamma}}}^R\|^2\right]}$$

with:

- $\bar{\boldsymbol{\gamma}}^R$: vector of the true RP whose i-th element, $\bar{\gamma}_i^R$, is the reflectivity of the scatterer associated to the i-th sub-bins,
- $\hat{\bar{\boldsymbol{\gamma}}}^R$: estimated RP.

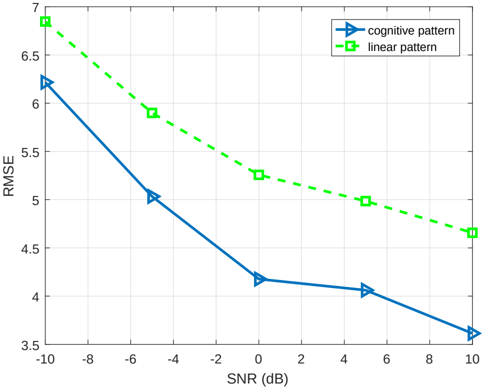
Its evaluation via **Monte Carlo technique** is considered in the following:

$$\overline{\text{RMSE}} = \sqrt{\frac{1}{M_c}\sum_{i=1}^{M_c}\|\bar{\boldsymbol{\gamma}}^R - \hat{\bar{\boldsymbol{\gamma}}}_i^R\|^2}$$

with $M_c = 100$ independent trials.

Performance Analysis

The RMSE is reported vs the SNR, exploiting the BIC-based IAA recovery strategy.



Conclusions

- A **HRRP recovery algorithm** has been designed exploiting the dynamic optimization of the probing waveform and some feedback information about the target RP.
- The transmitted waveform is **designed to minimize** the trace of the predicted CRLB matrix of the unknown parameters.
- An **efficient iterative procedure** has been developed to handle the NP-hard problem at hand.
- The results have shown that the cognitive paradigm allows to **enhance the RP estimation capabilities** in comparison with a conventional linear SF approach.
- **Future research tracks** might concern:
  - the analyses on real radar data.

THANK YOU FOR THE KIND ATTENTION