Implementation of a Flexible Frequency-Invariant Broadband Beamformer Based on Fourier Properties

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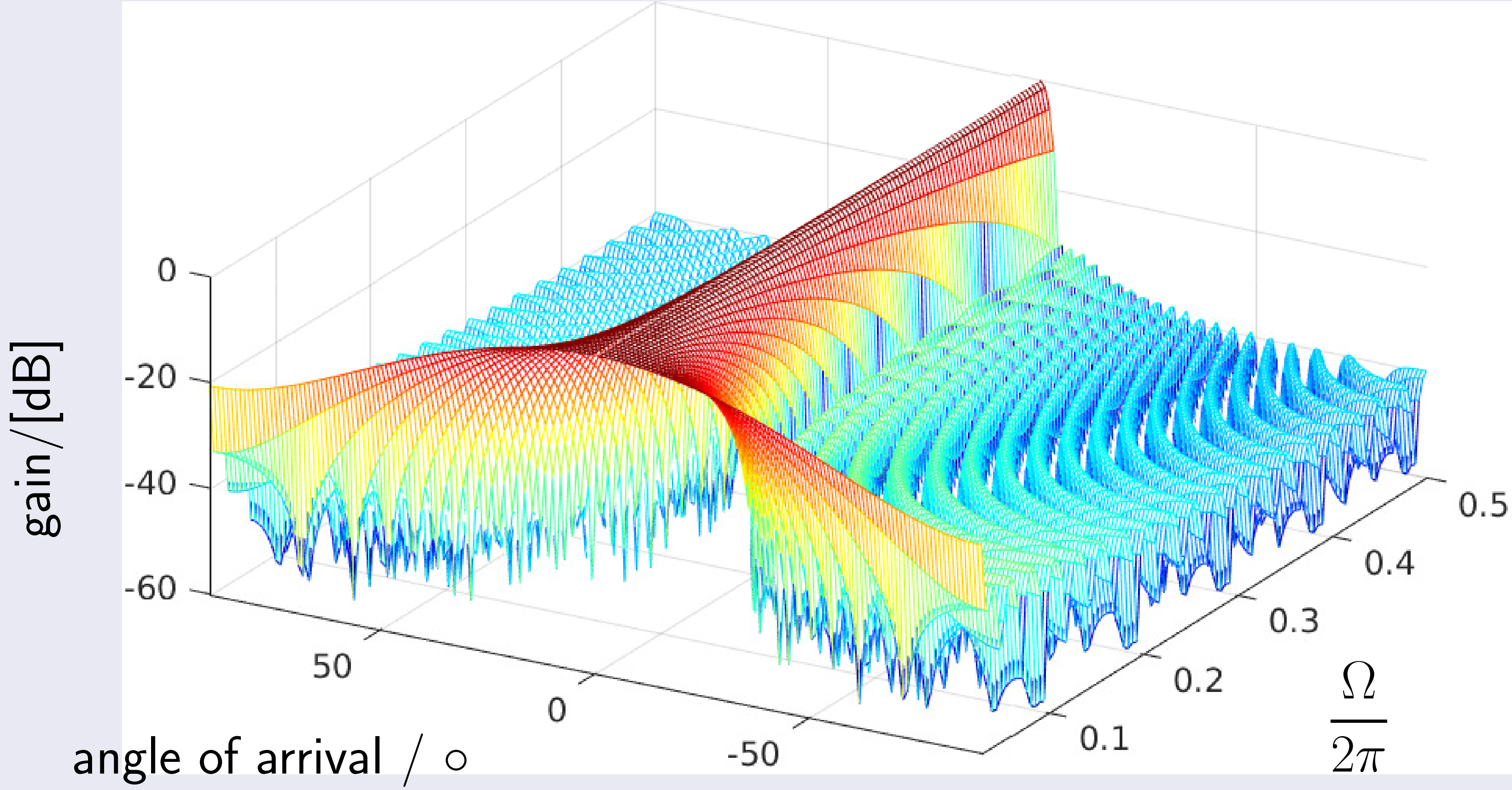
Broadband Beamforming

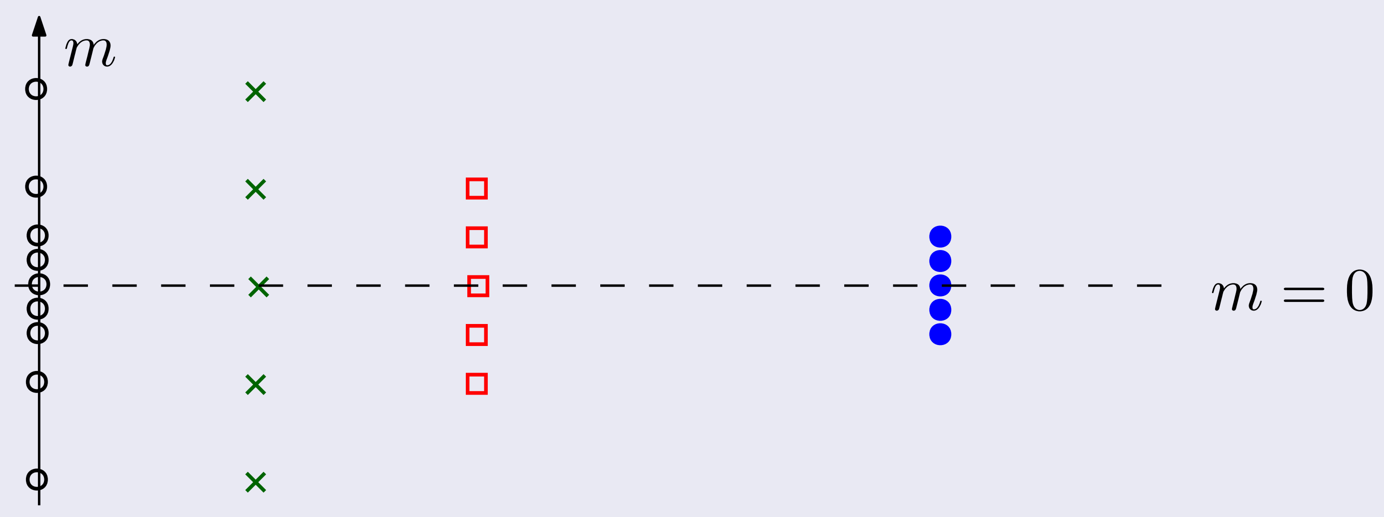
- The beamwidth of an array is proportional to its aperture, and inversely proportional to frequency; in an array of fixed size, beamwidth is therefore frequency-dependent:



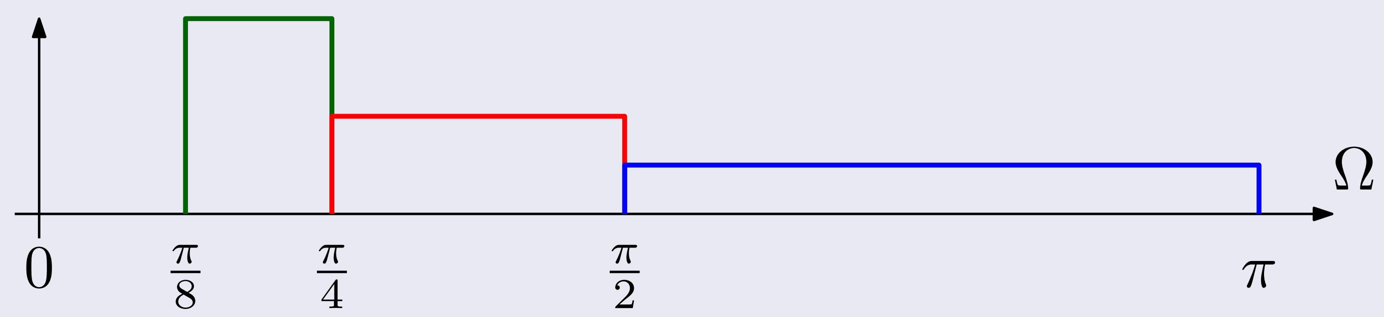
- frequency-invariant solutions have been suggested, see e.g. [1,2];
- generally: the aperture is controlled in a frequency-dependent fashion, adopting the array's widest beamwidth (i.e. poorest resolution) across the operational bandwidth;
- we discusss the combination of a nested array [2] with an implementationally simple frequency-invariant design.

Octave-Invariant Array Through Nesting

- We select a non-uniformly spaced nested array, which comprises of subarrays with dyadically scaled aperture but otherwise identical number of elements and configurations:



- each subarray processes data within a different octave: with every doubling of frequency, we halve the subarray's aperture [Trees & Bell, 2002]:



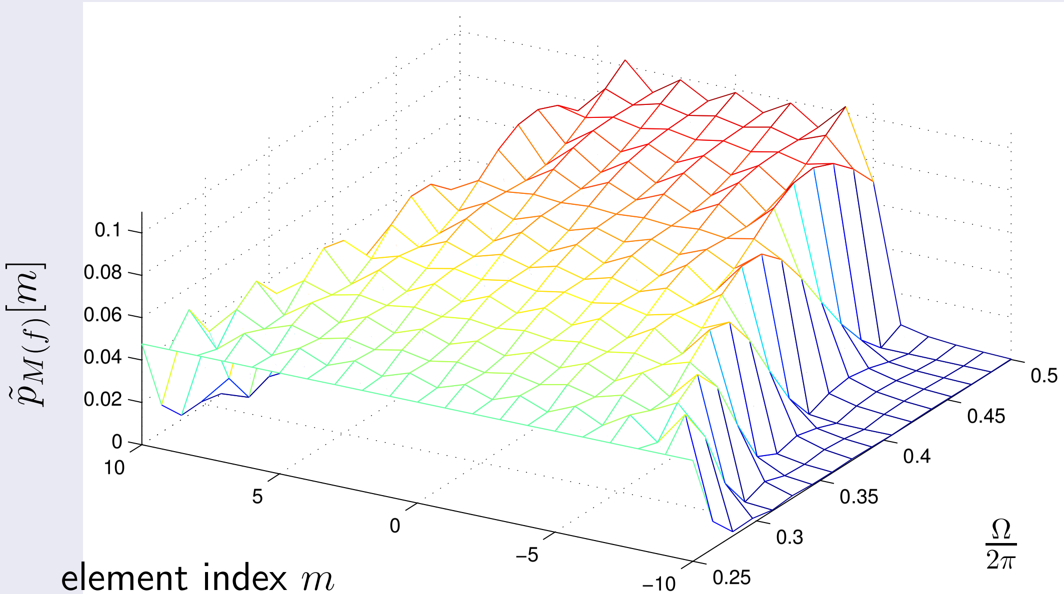
- using the same array processing, the overall array is invariant from octave to octave.

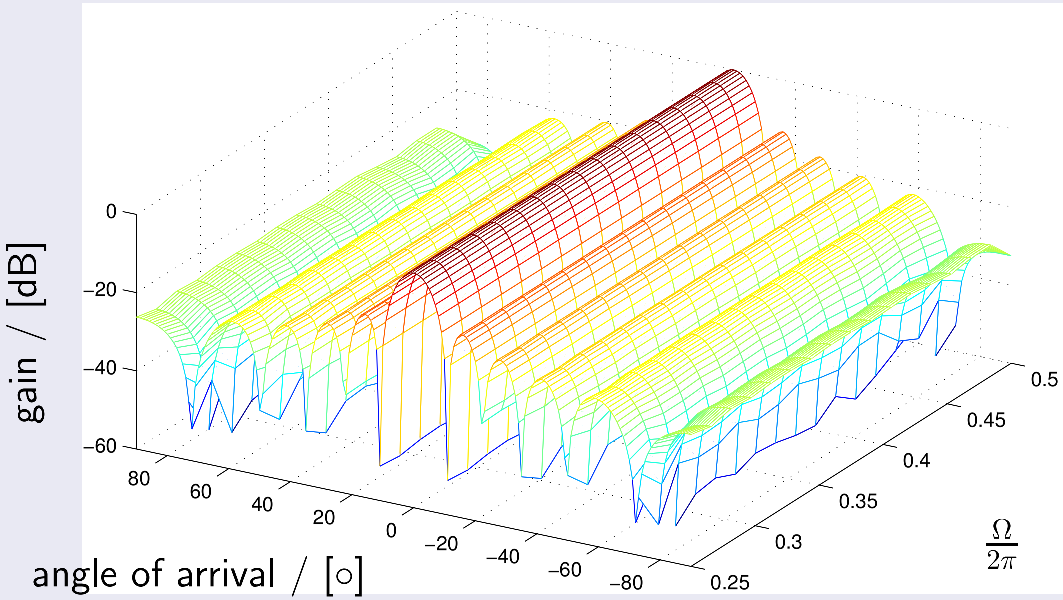
Frequency-Dependent Aperture

- A discrete rectangular array corresponds to a Dirichlet kernel in beamspace,

$$p_M[m] \circ\!\!-\!\!\bullet \frac{\sin \frac{M}{2}\Psi}{\sin \frac{1}{2}\Psi} \; ;$$

- we scale a fundamental period of the kernel inversely proportional to frequency and re-periodise;
- the inverse discrete Fourier transform over one octave yields





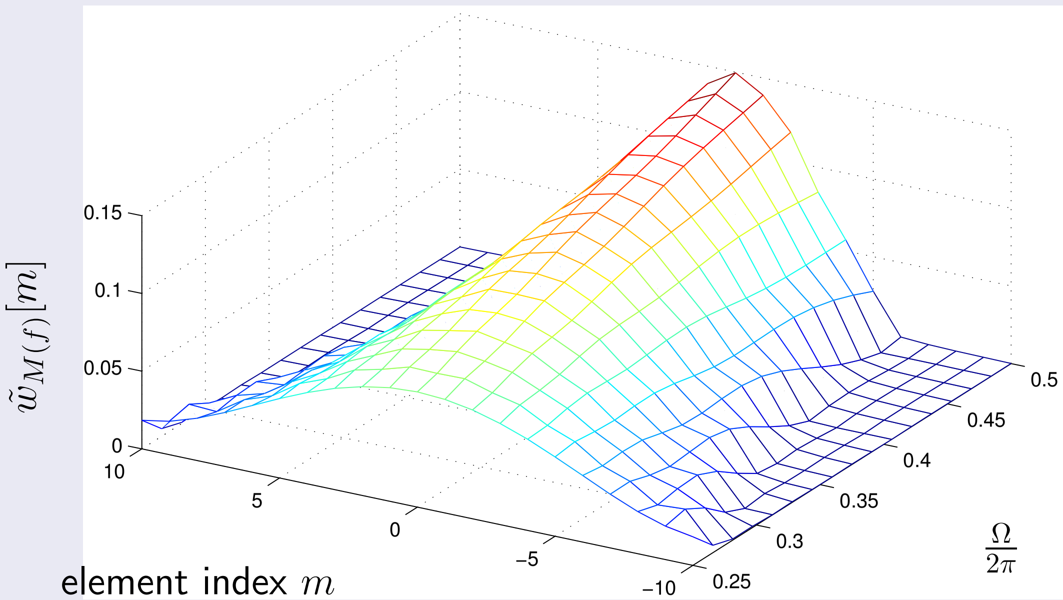
- note: aperture now decreases with frequency, leading to a constant beamwidth.

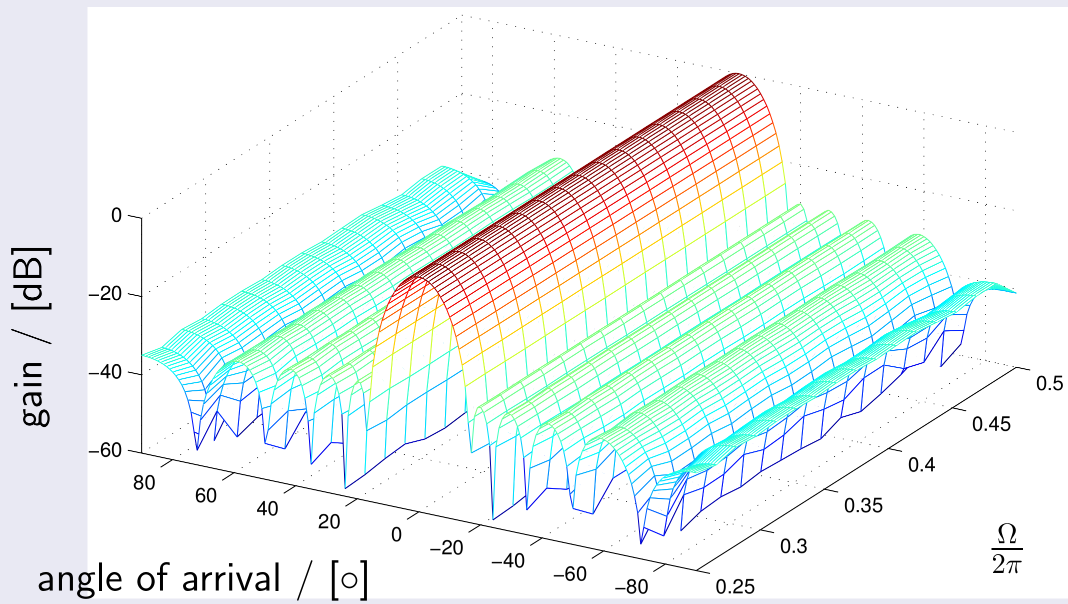
Arbitrary Windowing

- When applying an arbitrary window $w_M[n]$, the multiplication in the element domain corresponds to a convolution in beamspace;

$$p_M[m] \cdot w_M[m] \circ\!\!-\!\!\bullet \sum_{k=-K}^{K} \frac{\sin \frac{M}{2}(\Psi - \frac{2\pi k}{M})}{\sin \frac{1}{2}(\Psi - \frac{2\pi k}{M})} \cdot W(\mathrm{e}^{\mathrm{j}\frac{2\pi k}{M}}) \; ;$$

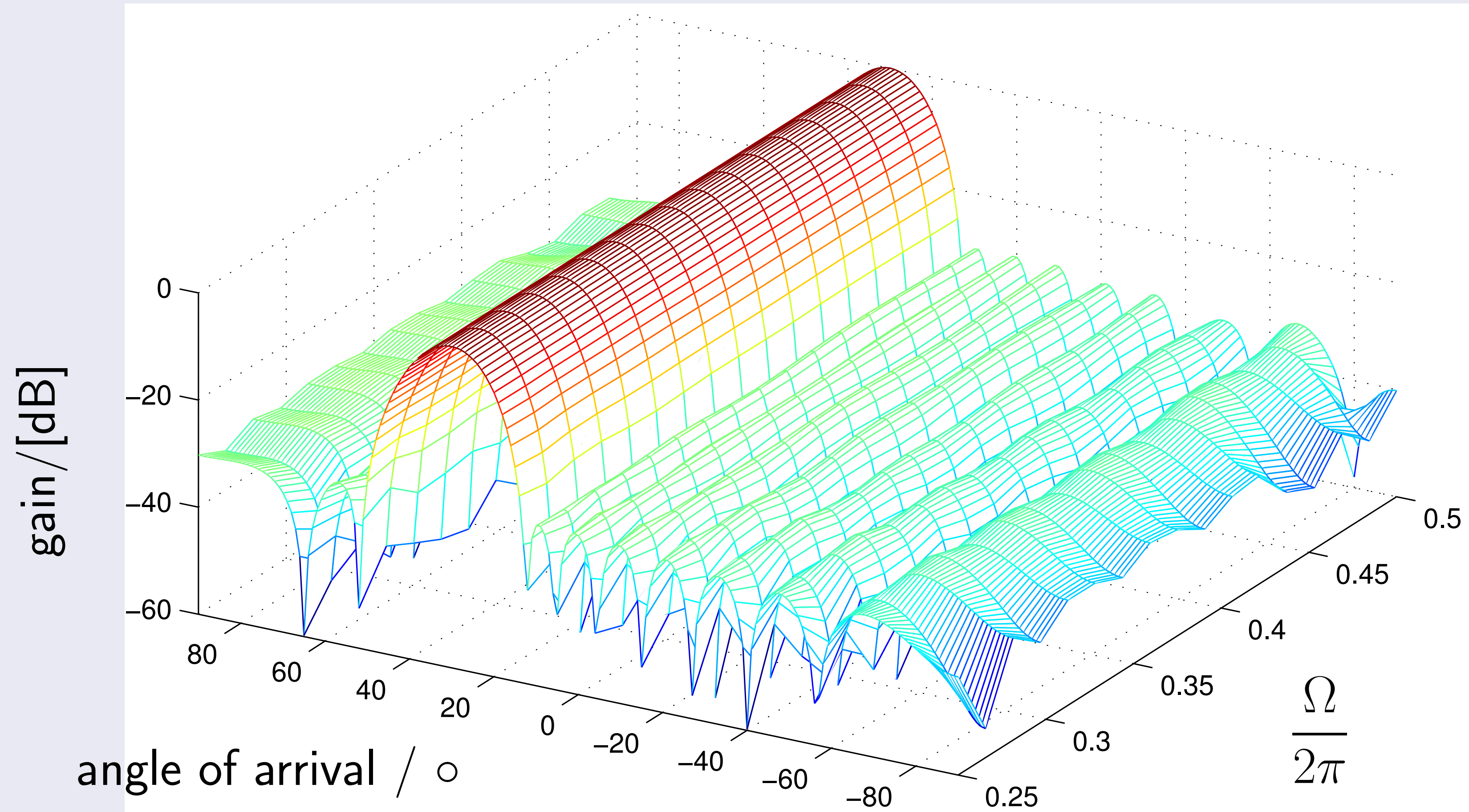
- most windows only have very few coefficients in beamspace (Hann window: $K=1$; Taylor window: usually $K=2$ sufficient for a very accurate implementation), hence the convolution is easily implemented;
- example for a Taylor window:





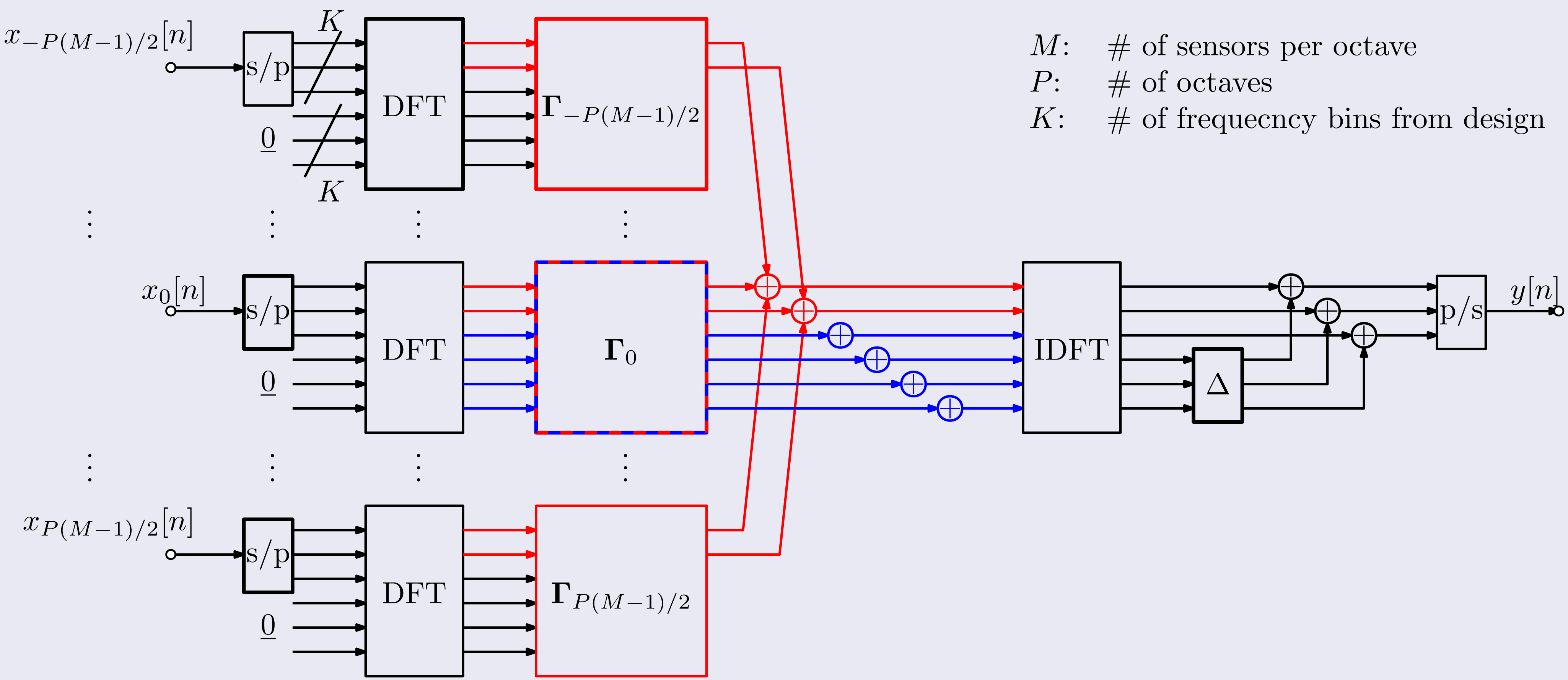
Look-Direction Selection

- For a look direction other than $\vartheta_0 = 0°$, it is possible to introduce a shift in beamspace;
- this corresponds to a modulation in element space;
- because this modulation is applied to a fundamental period before scaling and re-periodisation, discontinuities arise at the margins of the fundamental period — this is the only degradation;
- example for Taylor window, $M = 21$, with $\vartheta_0 = 30°$:



Implementation

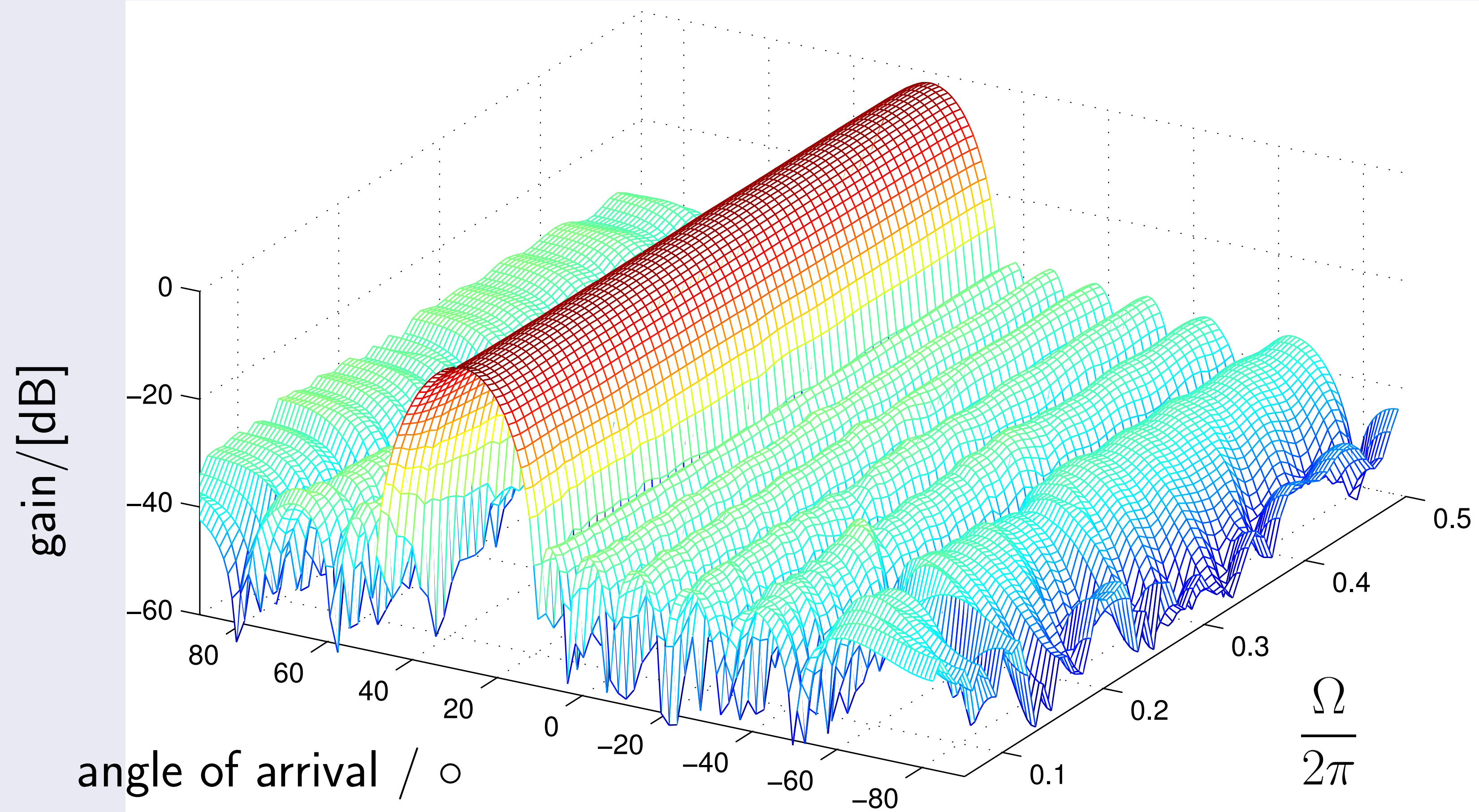
- The previous analysis has defined frequency-dependent element-space weights;
- an implementation therefore is performed in the discrete Fourier domain;
- an overlap-add implementation avoids mismatches between linear / cyclic convolutions;



- this requires a doubling of frequency points from the design;
- the element-space design is expanded by a factor of two in frequency, and interpolated by zeroing half the coefficients in the time domain before transforming back.

Results

- Nested array with $M = 21$ sensors in each of 4 octaves with a Taylor window pointing to $\vartheta = 30°$:



(Some) References

[1] W. Liu and S. Weiss: "Wideband Beamforming — Concepts and Techniques", Wiley, 2010.

[2] H.L. Van Trees: "Detection, Estimation and Modulation Theory: Optimum Array Processing", Wiley, 2002.