Co-prime Arrays with Reduced Sensors (CARS) for Direction-of-Arrival Estimation

Mingyang Chen[1], Lu Gan[2] and Wenwu Wang[1]

[1]Centre for Vision Speech and Signal Processing, University of Surrey
[2]Department of Electronic and Computer Engineering, Brunel University

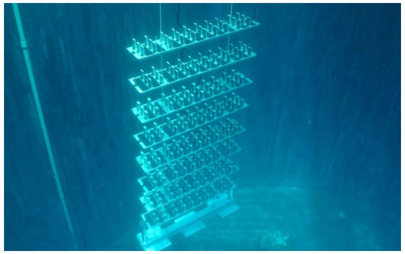
EPSRC
Engineering and Physical Sciences Research Council
[dstl]
UDRC

Sensor Signal Processing for Defence (SSPD2017)

Contents

- Motivation
- Background
  - Signal Model
  - Nested Array
  - Co-prime Arrays
- Co-prime Arrays with Reduced Sensors (CARS)
- Summary and Future work

Array System

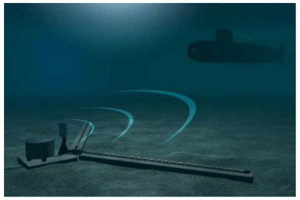


Acoustic array system

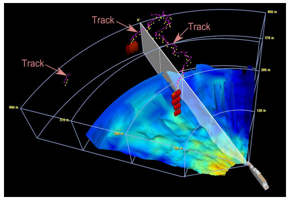
Antenna array system

Telescope array system

Applications:

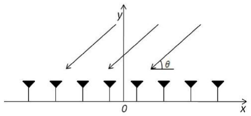


Underwater detection

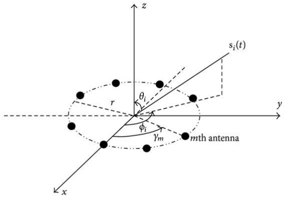


Target tracking

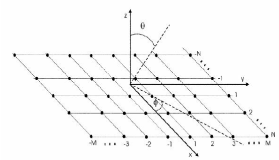
Direction of Arrival (DoA) Estimation



Linear array



Circular array

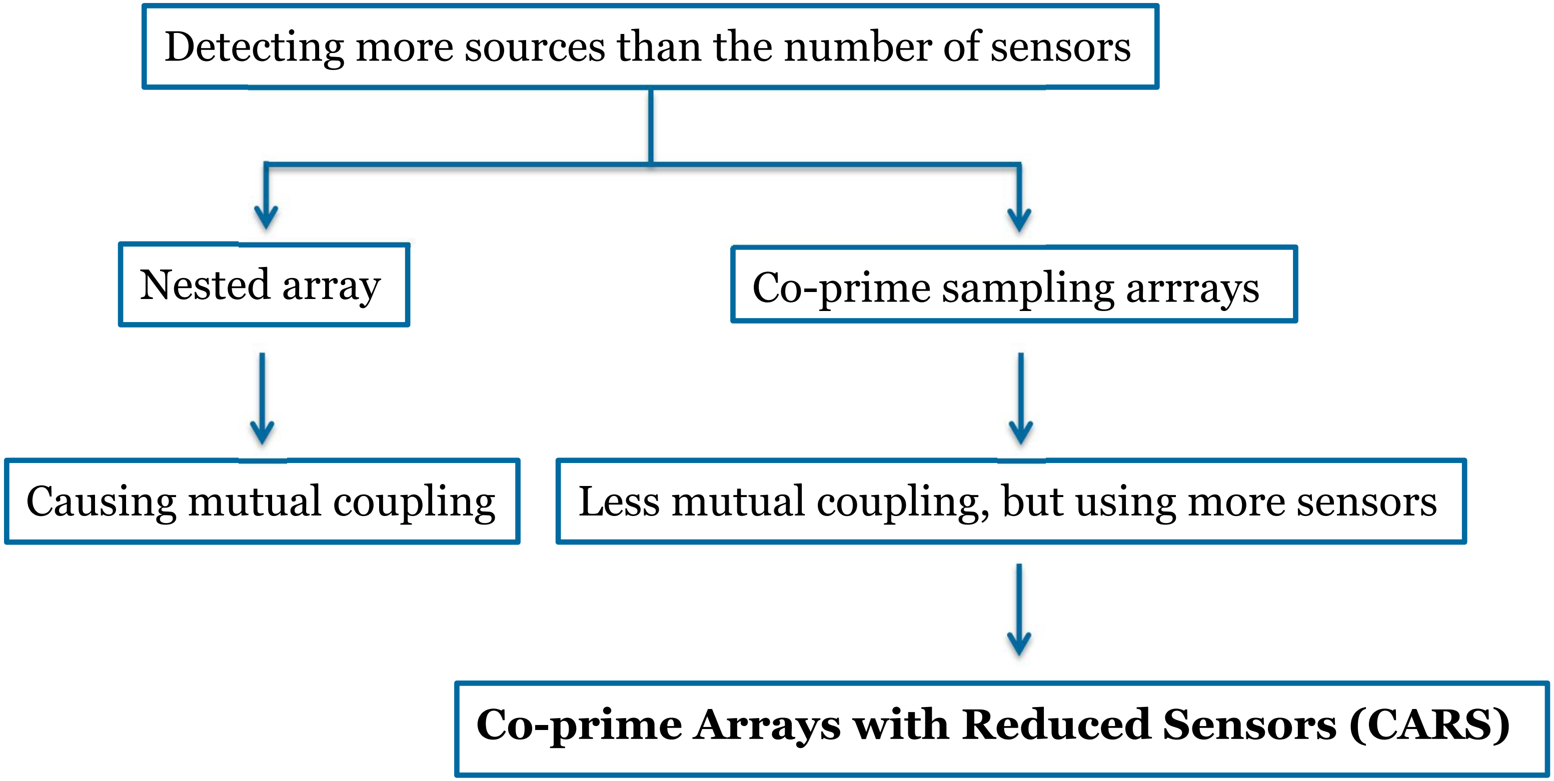


Planar array

Problem:

How to detect more sources than the number of sensors?

Motivation


Detecting more sources than the number of sensors
Nested array
Co-prime sampling arrrays
Causing mutual coupling
Less mutual coupling, but using more sensors
Co-prime Arrays with Reduced Sensors (CARS)

UNIVERSITY OF SURREY

Signal model

$$[y(1), y(2), \cdots, y(K)] = \mathbf{AX} + \Omega$$

S is the total number of sensors.

D is the number of narrowband far-field sources arriving in one half of the plane.

K is defined as snapshot.

$\mathbf{A} : [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots, \mathbf{a}(\theta_D)] \in \mathbb{C}^{S \times D}$

$\mathbf{a}(\theta_i) : [1, e^{j2\pi\theta_i Sensor_2}, \cdots, e^{j2\pi\theta_i Sensor_S}]^T \in \mathbb{C}^S$

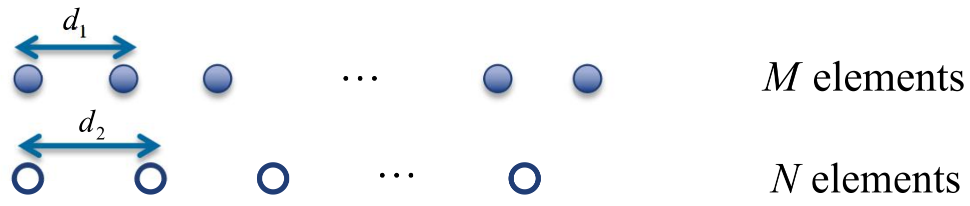
$\mathbf{X} : [\mathbf{x}(1), \mathbf{x}(2), \cdots \mathbf{x}(K)] \in \mathbb{C}^{D \times K}$

$\Omega : [\mathbf{n}(1), \mathbf{n}(2), \cdots \mathbf{n}(K)] \in \mathbb{C}^{S \times K}$

$\mathbf{n}(k) \in \mathbb{C}^S$ is assumed to be independent and identically distributed (i.i.d) random noise vector.

$\theta_i : (d / \lambda) \sin\theta_i$ is the normalized DoA.

Difference co-array



For this pair of uniform linear arrays (ULA), the sensors are positioned at

$$\mathrm{P} = \{Md_1\} \bigcup \{Nd_2\}$$

The maximum number of the difference lags is determined by the number of unique elements in the following set

$$\mathrm{L}_p = \{l_p \mid l_p \lambda / 2 = u - v, u \in \mathrm{P}, v \in \mathrm{P}\}$$

The difference co-array consists of either **self-differences** or **cross-differences**. The self-difference in the coarray has positions

$$\mathrm{L}_s = \{l_s \mid l_s = Md_1\} \bigcup \{l_s \mid l_s = Nd_2\}$$

whereas the cross-difference has positions

$$\mathrm{L}_c = \{l_c \mid l_c = Md_1 - Nd_2\}$$

UNIVERSITY OF SURREY

Degrees-of-Freedom (DoF)

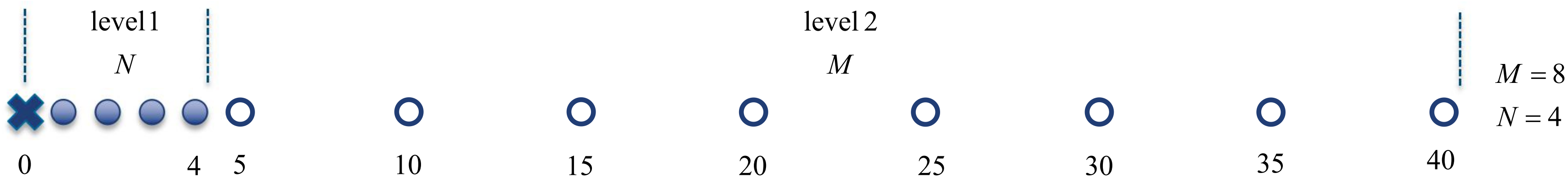
The Degrees-of-Freedom (DoF) here denotes the cardinality of the ULA segments in the difference co-array set, which consists of **differences between any pair of sensors** positions in the array structure.

Let the set $\mathbb{U}$ denote the maximum contiguous ULA segments in $\mathbb{L}_p$. The number of elements in $\mathbb{U}$ is called the number of **uniform degrees-of-freedom**.

The **uniform DoF** is required to implement MUSIC and ESPRIT algorithms.

Background

Nested array



The level 1 samples are at: $1 \le \ell \le N$

The level 2 samples are at: $(N+1)\delta,\ 1 \le \delta \le M$

The cross-differencec lie in the range: $-[(N+1)M-1] \le l_c \le [(N+1)M-1]$

The self-differences of the first array: $(N+1)(\delta_1-\delta_2),\ 1 \le \delta_1,\ \delta_2 \le M$

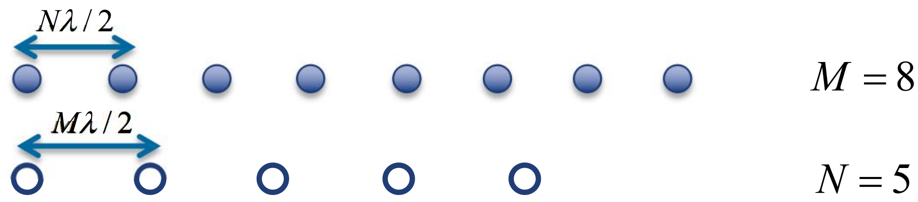
The self-differences of the second array: $-(N-1) \le l_s \le (N-1)$

The number of **uniform degrees-of-freedom**: $2[(N+1)M-1]+1 = 2(N+1)M-1$

In practice, any sensor output is influenced by its neighboring elements, which is called **mutual coupling**.

UNIVERSITY OF SURREY

Co-prime arrays



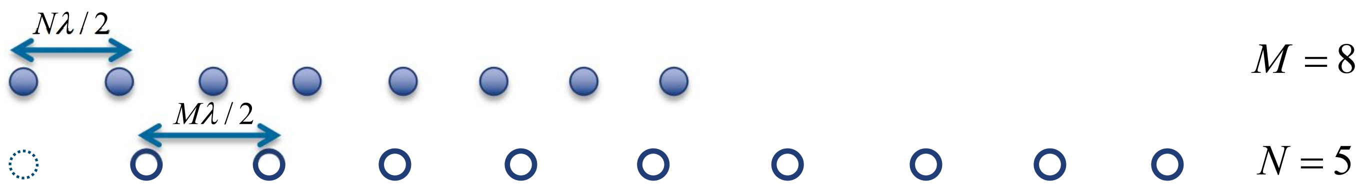
In this array configuration, the **self-differences** of the two subarrays are given by

$$\mathrm{L}_s = \{l_s \mid l_s = Mn\} \cup \{l_s \mid l_s = Nm\}, \ \ 0 \le n \le N-1, \ 0 \le m \le M-1$$

The **cross-differences** between the two subarrays are given by

$$\mathrm{L}_c = \{l_c \mid l_c = Mn - Nm\}$$

Improved co-prime arrays [Piya Pal, et al. 2011]:



Difference co-array: $Mn - Nm, \ 1 \le n \le 2N-1, \ 0 \le m \le M-1$

The number of sensors: $M + 2N - 1$

The number of uniform degrees-of-freedom: $2MN + 1, \ -MN \le L_p \le MN$

UNIVERSITY OF SURREY

Improved co-prime arrays vs Nested array

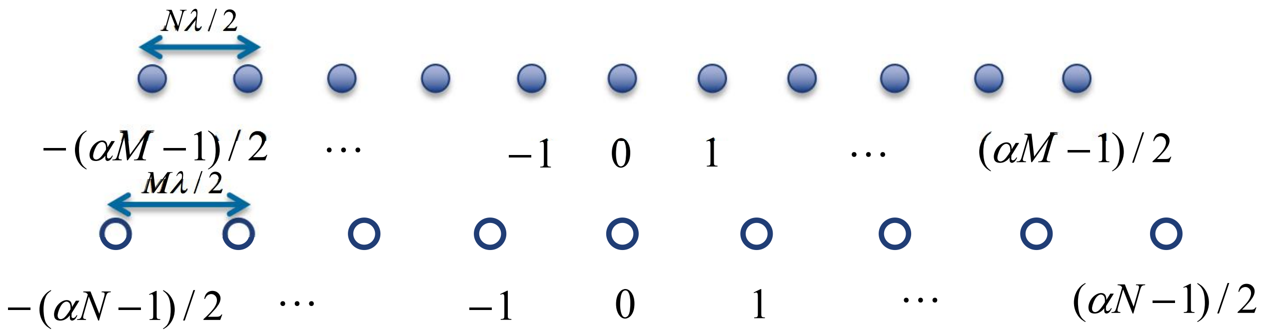
	Improved Co-prime Arrays	Nested Array
No. of Sensors	$M+2N-1$	$M+N$
Uniform DoF	$2MN+1$	$2MN+1$
No. of ULAs	2	1
Inter-element Spacing	$N\lambda/2$, $M\lambda/2$	$\lambda/2$, $(N+1)\lambda/2$
Mutual Coupling	Relatively small	Significant

Compared to the nested arrays, the co-prime array structure mitigates mutual coupling, however, it uses more sensors to attain the same uniform DoF.

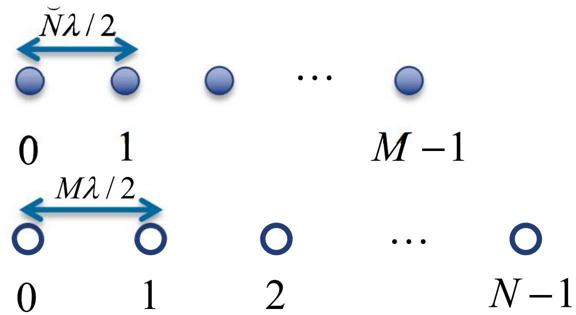
UNIVERSITY OF SURREY

Some other co-prime array structures

Center symmetric co-prime arrays [Yang Liu, et al. 2016]:



Co-prime array with compressed inter-element spacing (CACIS) [Si Qin, et al. 2015]:



$M > N$

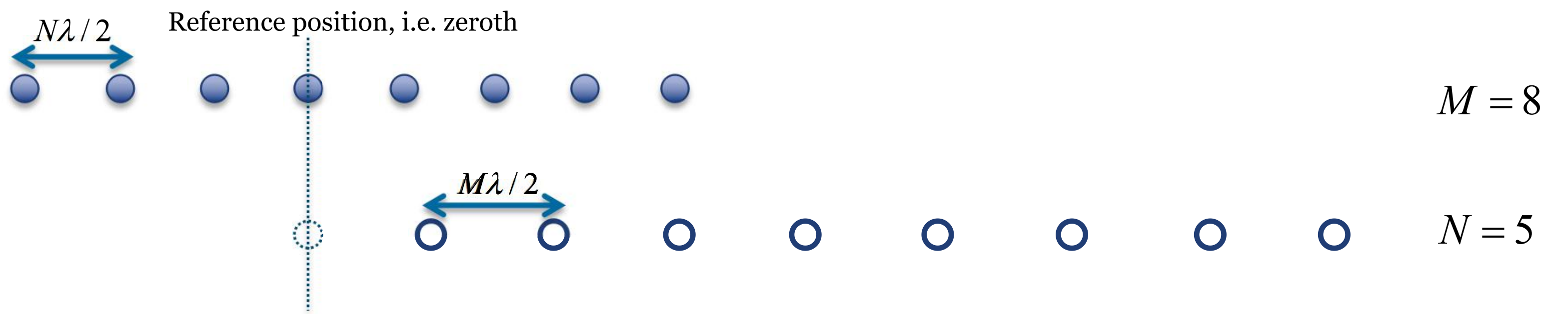
$N = \beta\breve{N}$

It is necessary to explore the **sensor reduction strategy** for co-prime arrays.

Co-prime Arrays with Reduced Sensors (CARS)

UNIVERSITY OF SURREY

Sensor reduction strategy for co-prime arrays:



The sensors in CARS are **located** at:

$$\mathrm{P}=\{Mn\lambda/2 \mid 1\le n\le 3N/2 \text{ when } N \text{ is even}, 1\le n\le 3N+1/2 \text{ when } N \text{ is odd}\}\cup$$
$$\{Nm\lambda/2 \mid -M/2+1\le m\le M/2 \text{ when } N \text{ is even}, -(M-1)/2\le m\le (M-1)/2 \text{ when } N \text{ is odd}\}$$

The **uniform DoF** of CARS structure is as follows:

(1) When M is odd, N is even: $2MN+N+2M-1$

(2) When M is even, N is odd: $2MN+3M-1$

(3) When M is odd, N is odd: $2MN+N+3M-1$

UNIVERSITY OF SURREY

Proof of the achieved uniform DoF:

We consider the situation when M is even, N is odd.

Given

$$0 \le l_c \le MN + 3M/2 - 1$$

Since

$$-M/2 + 1 \le m \le M/2 \Rightarrow -MN/2 + N \le Nm \le MN/2$$

and

$$l_c = Mn - Nm \Rightarrow Mn = l_c + Nm$$

We have

$$-MN/2 + N \le Mn \le 3MN/2 + 3M/2 - 1$$

Since M and N are integers, we get

$$-MN/2 + N \le Mn < 3M(N+1)/2 \Rightarrow -N/2 + N/M \le n < 3(N+1)/2$$

When $n < 0, Mn < 0$. If $l_c = Mn - Nm > 0, m < 0$. As $N(-m) - M(-n) = N\breve{m} - M\breve{n}$, which can be regarded as the flipped positive values in $Mn - Nm$. So we only need to consider $n \ge 0$ and obtain

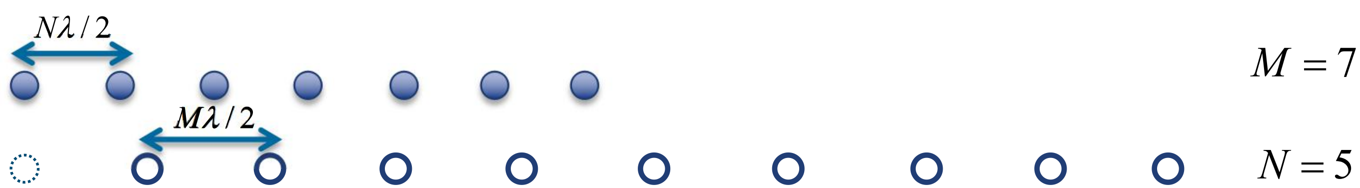
$$1 \le n \le (3N+1)/2$$

which is satisfied in the proposed co-prime arrays.

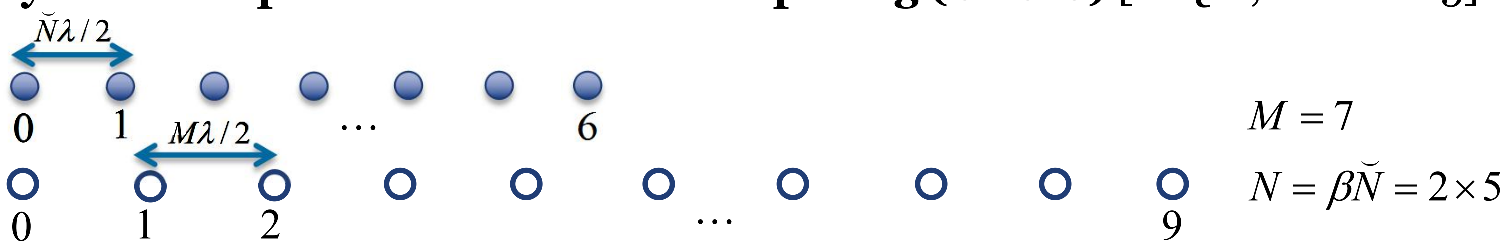
CARS vs Other Co-prime Arrays

Co-prime Arrays with Reduced Sensors (CARS):

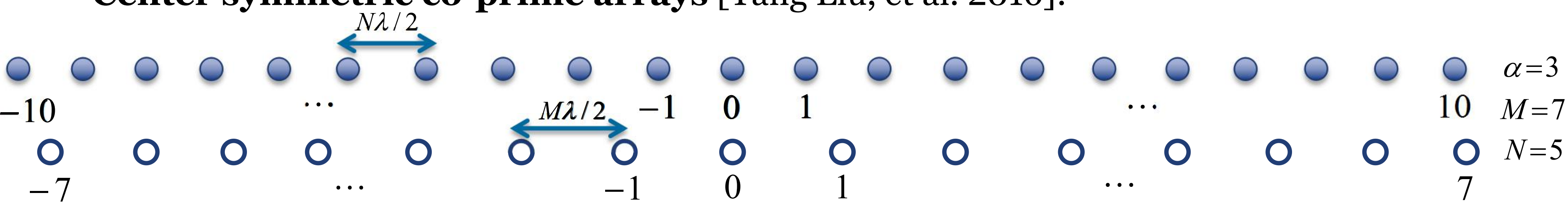
Improved co-prime arrays [Piya Pal, et al. 2011]:



Co-prime array with compressed inter-element spacing (CACIS) [Si Qin, et al. 2015]:



Center symmetric co-prime arrays [Yang Liu, et al. 2016]:



CARS vs Other Co-prime Arrays

	When M is odd, N is even.	When M is even, N is odd.	When M is odd, N is odd.
CARS	No. of sensors: M+3N/2 Uniform DoF: 2MN+N+2M-1	No. of sensors: M+(3N+1)/2 Uniform DoF: 2MN+3M-1	No. of sensors: M+(3N+1)/2 Uniform DoF: 2MN+N+3M-1
Improved co-prime arrays	No. of sensors: M+2N-1 Uniform DoF: 2MN+1	No. of sensors: M+2N-1 Uniform DoF: 2MN+1	No. of sensors: M+2N-1 Uniform DoF: 2MN+1
CACIS (compressed factor is 2)	No. of sensors: M+2N-1 Uniform DoF: 2MN+2N-1	No. of sensors: M+2N-1 Uniform DoF: 2MN+2N-1	No. of sensors: M+2N-1 Uniform DoF: 2MN+2N-1
Center symmetric co-prime arrays (extension factor is 2)	-	-	No. of sensors: 2M+2N Uniform DoF: 2MN+M+N-1

The proposed CARS structure can get a reduction of about $floor\left(\frac{N}{2}\right)$ sensors with increased uniform degrees-of-freedom.

UNIVERSITY OF SURREY

Multiple Signal Classifier (MUSIC) Algorithm

The covariance matrix of data vector $\mathbf{y}$ is obtained as

$$R_{yy} = \sum_{i=1}^{D} \sigma_i^2 \mathbf{a}(\theta_i)\mathbf{a}^H(\theta_i) + \sigma^2 \mathbf{I}$$

where σ_i^2 is the power of the i-th signal, σ^2 is the noise power.

We vectorize $R_{\mathbf{yy}}$ to obtain the following vector

$$\mathbf{z} = \mathrm{vec}(R_{yy}) = \widetilde{\mathbf{A}}\mathbf{f} + \sigma_s^2 \widetilde{\mathbf{I}}_s$$

where $\widetilde{\mathbf{A}} = [\mathbf{a}^*(\theta_1) \otimes \mathbf{a}(\theta_1), \cdots, \mathbf{a}^*(\theta_D) \otimes \mathbf{a}(\theta_D)]$, $\mathbf{f} = [\sigma_1^2, \sigma_2^2, \cdots, \sigma_D^2]^T$, $\widetilde{\mathbf{I}}_s = [\mathbf{e}_1^T, \mathbf{e}_2^T, \cdots, \mathbf{e}_S^T]$ with $\mathbf{e}_s$ being a column vector of all zeros except value 1 at the s-th position.

UNIVERSITY OF SURREY

Spatial smoothing based rank enhancement [Piya Pal, et al. 2011]

Denote $[-r, r]$ as the consecutive lag range, a new vector $\mathbf{z}_1$ is given by

$$\mathbf{z}_1 = \mathbf{A}_1\mathbf{f} + \sigma_s^2 \widetilde{\mathbf{I}}_1$$

where $\mathbf{A}_1$ is a new matrix of size $(2r+1)\times D$ from $\widetilde{\mathbf{A}}$.

$\widetilde{\mathbf{I}}_1$ is a $(2r+1)\times 1$ vector of all zeros except value 1 at the $(r+1)$-th position.

Dividing this re-built array into $(r+1)$ overlapping subarrays, of which contains $(r+1)$ elements.

Define

$$\mathbf{R}_i = \mathbf{z}_{1i}\mathbf{z}_{1i}^H$$

Taking the average of $\mathbf{R}_i$ over all i, we obtain

$$\mathbf{R}_{ss} = \frac{1}{r+1}\sum_{i=1}^{r+1}\mathbf{R}_i$$

Spatial smoothing works only for a continuous set of differences.

UNIVERSITY OF SURREY

Experiment setup

- Narrowband DoA estimation through a pair of co-prime linear arrays.
- 35 stationary simulated sources with the DoA profiles of

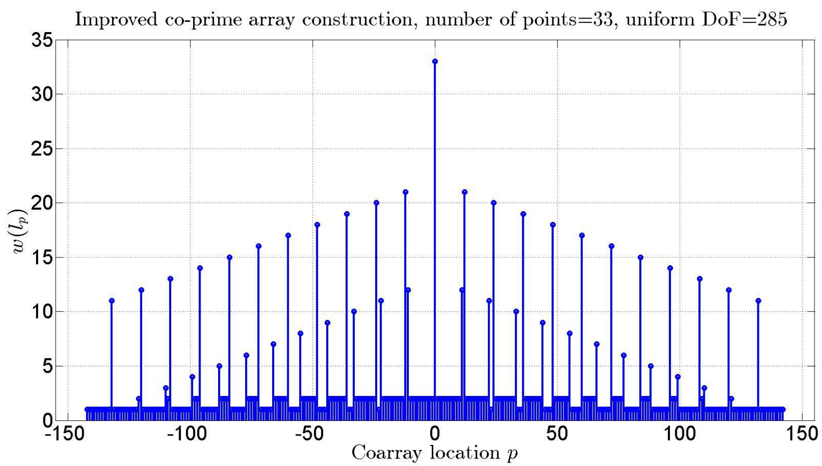
$$\tilde{\theta}_i = -0.1 + 0.2 \times (i-1)/35, \; i = 1, 2, \cdots, 35$$

- Signal to Noise Ratio (SNR) is 5 dB.
- The number of snapshots $K = 800$.
- M is chosen to be 12 and N is 11.
- The associated MUSIC spectra $P(\tilde{\theta})$.
- The root-meansquared error (RMSE)

$$Error = \sqrt{\frac{1}{D}\sum_{i=1}^{D}(\hat{\tilde{\theta}}_i - \tilde{\theta}_i)^2}$$

where $\hat{\tilde{\theta}}_i$ denotes the estimated normalized DoA of the i-th source signal and $\tilde{\theta}_i$ is the designed normalized DoA.

Co-prime Arrays with Reduced Sensors (CARS)



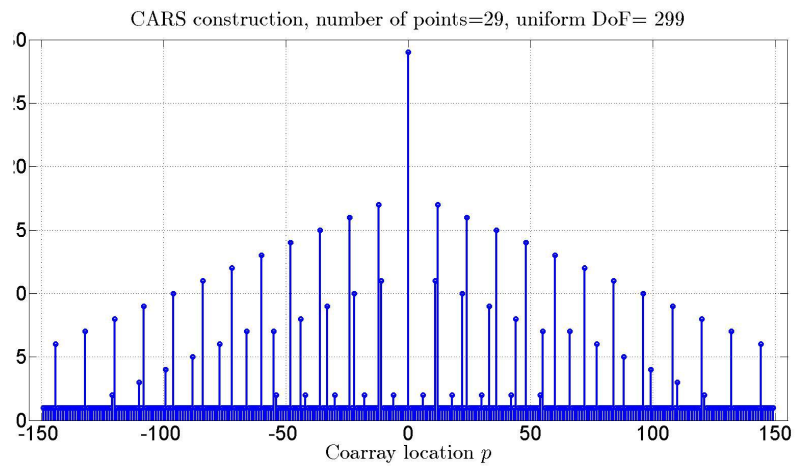


Fig. 1. The values of **weight function** and **the maximum contiguous segments**.

- The weight function $\omega(l_p), l_p \in L_p$ of an array is defined as the number of sensor pairs which has the same value of coarray index l_p.
- **Improved co-prime arrays:** 33 sensors in total, uniform DoF is 285.
- **CARS structure**: 29 sensors, a total of 299 uniform DoF.

Co-prime Arrays with Reduced Sensors (CARS)

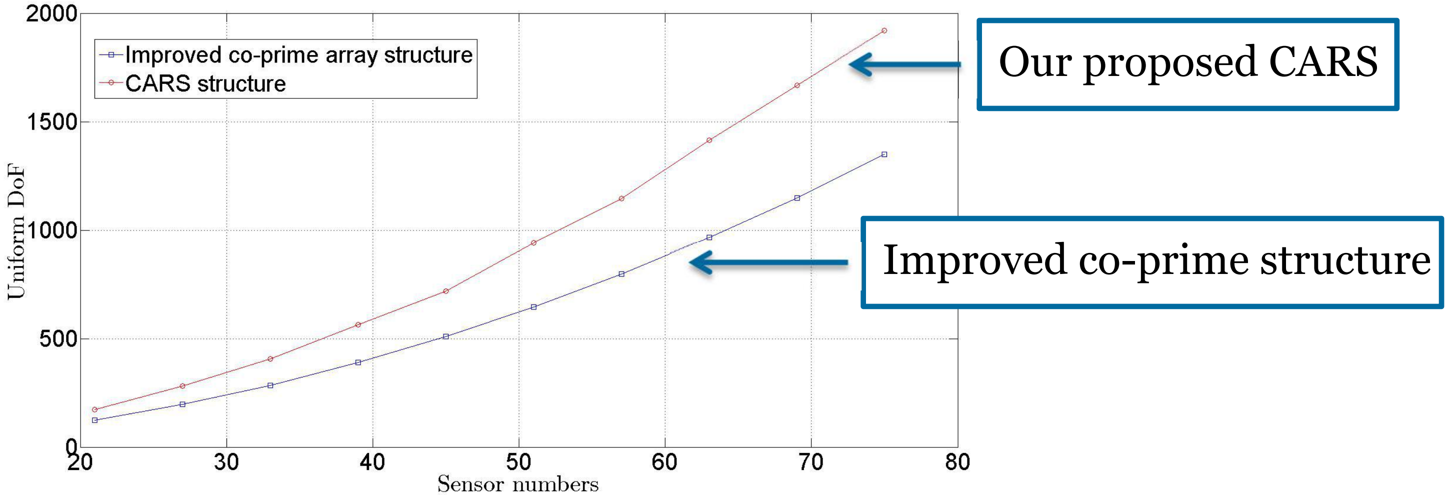


Fig. 2. Number of sensors vs uniform DoF.

- A comparison in uniform DoF between CARS and improved co-prime arrays.
- Considering using 21, 27, 33, 39, 45, 51, 57, 63, 69, 75 sensors.
- When using same number of sensors, the proposed CARS can achieve more number of uniform DoF.

Co-prime Arrays with Reduced Sensors (CARS)

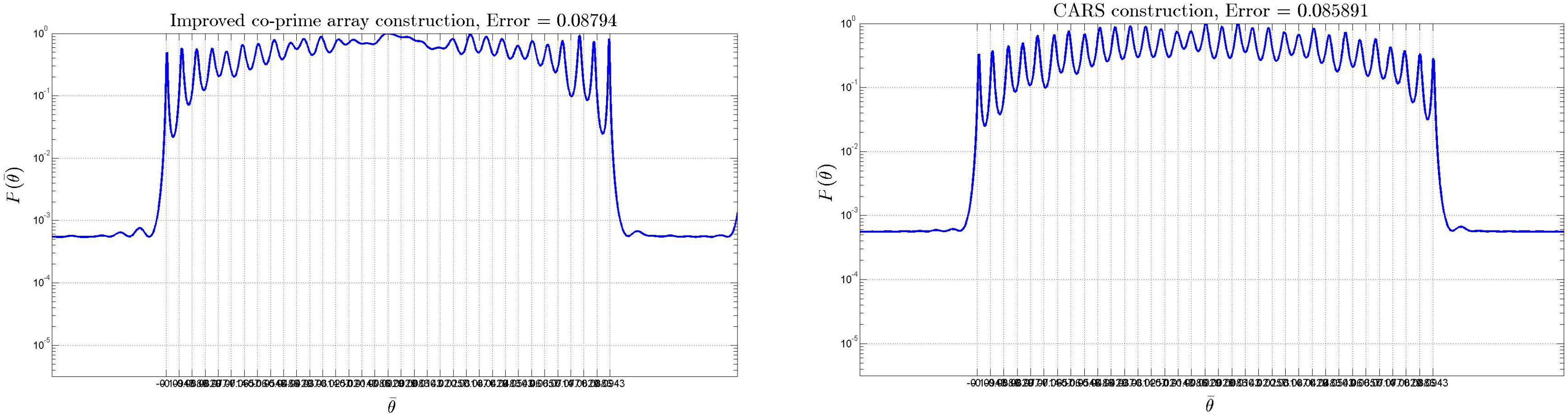


Fig. 3. The associated **MUSIC spectra** for the DoAs estimation.

We found that the proposed CARS structure gives **good DoA estimations** when the number of sources is larger than the number of sensors.

Summary

- The Co-prime Arrays with Reduced Sensors (CARS) has been presented to exploit the **co-array distribution** for source localisation.
- The structure contains a pair of co-prime subarrays, where the **first array** is shifted until approximately symmetrical to the center (reference sensor) and the number of sensors in the **second array** is set according to the odd and even of the co-prime number pair.
- The CARS structure achieves **more uniform DoF** than previous work with a **reduction in** the number of physical **sensors**.
- The **DoA estimation** results by the MUSIC algorithm evaluated for multiple sources with noise show **good performance** of the proposed CARS structure.

Future work

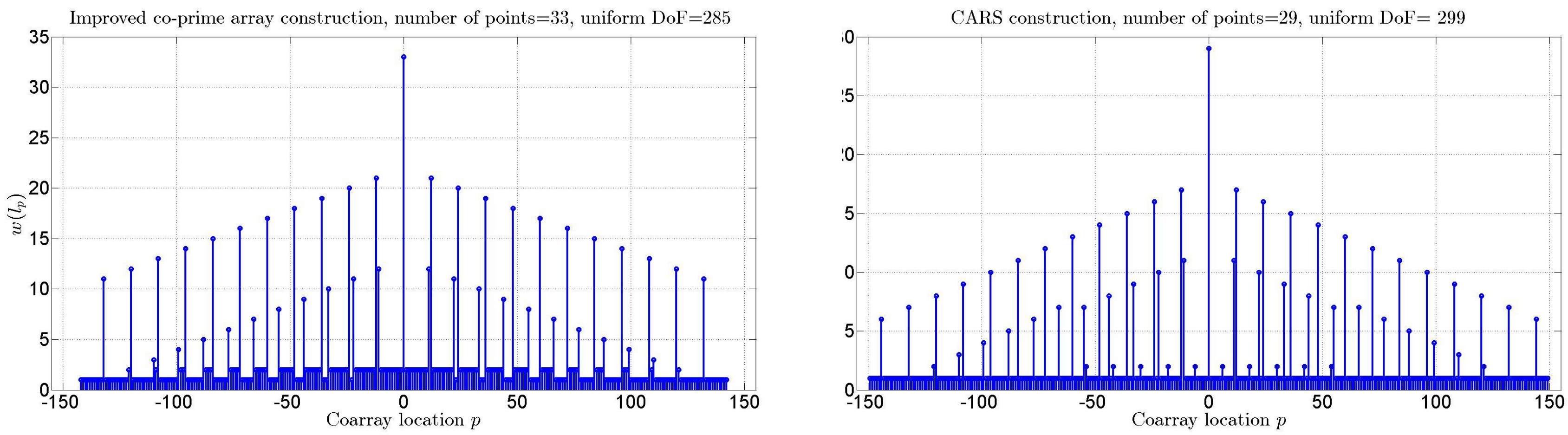


Fig. 1. The values of **weight function** and **the maximum contiguous segments**.

- **Sensor optimisation**: decreasing **weight function** reduces the **mutual coupling**, which implies the mutual coupling matrix is closer to the identity matrix, and makes the RMSE likely to decrease.
- **DoA estimation**: spatial sparsity based, correlated signals, wideband signals.

End

Thank you