Maximum likelihood signal parameter estimation via track before detect

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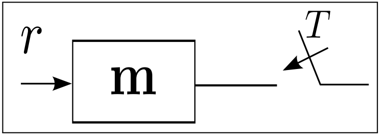
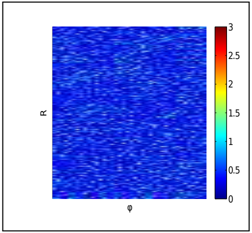
Introduction

- Active sensors locate objects by transmitting pulses and finding the range and angle of reflectors using received echos.
- The received signal is filtered and sampled, then each sample is tested to be identified as noise, or a reflection.
- In this work, we are interested in finding the parameters that describe these samples.





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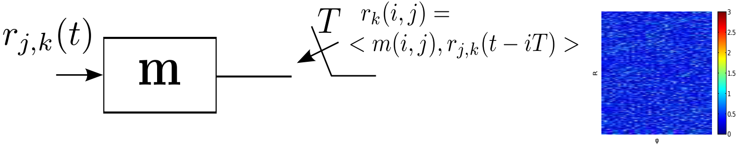
Problem Definition

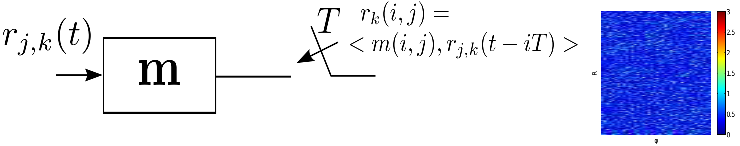


- The sampled complex signal is modelled by

$$r_k(i,j) = \begin{cases} Ee^{j\theta_k} + n, & H_1 : \text{If a reflector exists.} \\ n, & H_0 : \text{otherwise.} \end{cases}$$

  where $n \sim \mathcal{CN}(0, \beta^2)$.
- The SNR is $10 \log_{10} E^2 / beta^2$.
- The modulus $z_k(i,j) \triangleq |r_k(i,j)|$ is as the intensity value in the range-bearing bins.

Problem Definition



- Conventional algorithms, e.g., constant false alarm rate (CFAR) detectors, require β^2.
- Tracking algorithms require the probability of detection of targets which can be calculated only if E is known.
- Track-before-detect algorithms often require both model parameters.

ML Signal Parameter Estimation

- The maximum likelihood solution is given by

$$(\hat{E}, \hat{\beta}^2) = \arg\max_{E,\beta^2} \log l(\mathbf{z}_1, \cdots, \mathbf{z}_k | E, \beta^2).$$

- The log-likelihood has a recursive form

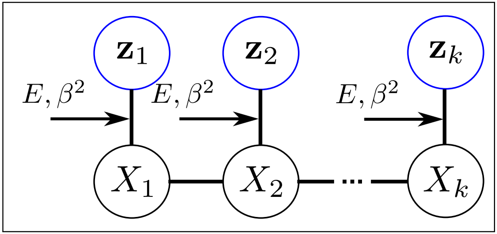
$$\log l(\mathbf{z}_{1:k}|E,\beta^2) = \log l(\mathbf{z}_{1:k-1}|E,\beta^2) + \log \underbrace{p(\mathbf{z}_k|\mathbf{z}_{1:k-1},E,\beta^2)}_{\text{The log-likelihood update term}}$$

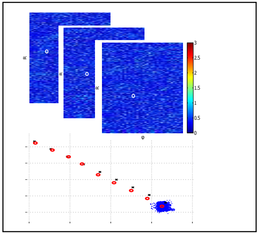
- Most of the algorithms in the literature (e.g., CFAR algorithms for estimating β^2) correspond roughly to the assumptions that the update term factorises
  1. without $\mathbf{z}_{1:k-1}$ incorporated into the update at k, i.e., $p(\mathbf{z}_k|\mathbf{z}_{1:k-1},E,\beta^2) = p(\mathbf{z}_k|E,\beta^2)$.
  2. using "hard decisions" to split the bins in $\mathbf{z}_k = \mathbf{z}_{k,noise} \cup \mathbf{z}_{k,target}$, i.e., $p(\mathbf{z}_k|E,\beta^2) = p(\mathbf{z}_{k,noise}|\beta^2)p(\mathbf{z}_{k,target}|E,\beta^2)$

Proposed Solution (1/4)

- A Bernoulli random finite set (RFS) model allows us to have a single Hidden Markov Chain, and, a joint likelihood:

$$X_k = \begin{cases} X_k = \{x_k\}, x_k \sim s_k(x_k), & \text{with prob. } r_k \\ X_k = \emptyset, & \text{with prob. } 1 - r_{k|k-1}. \end{cases} \quad (1)$$

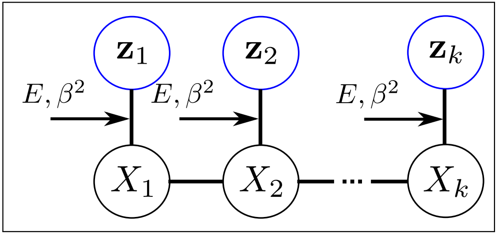


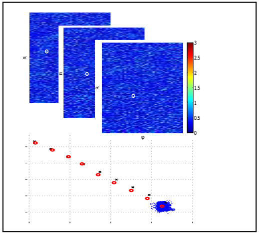


Proposed Solution (2/4)

- Facilitates a joint likelihood that incorporates the information from time 1 to k:

$$p(\mathbf{z}_k|\mathbf{z}_{1:k-1}, E, \beta^2) = \int p(\mathbf{z}_k|X_k, E, \beta^2)p(X_k|\mathbf{z}_{1:k-1}, E, \beta^2)\delta X_k \quad (2)$$



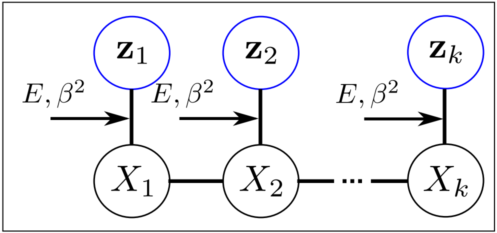


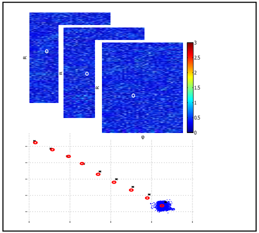
Proposed Solution (3/4)

- The score function (at E, β^2) is also recursive and derived in the article

$$\nabla \log l(\mathbf{z}_{1:k}|E,\beta^2) = \nabla \log l(\mathbf{z}_{1:k-1}|E,\beta^2) + \nabla \log p(\mathbf{z}_k|\mathbf{z}_{1:k-1},E,\beta^2)$$

- The Bernoulli track-before-detect filter in the article evaluates $\nabla \log p(\mathbf{z}_k|\mathbf{z}_{1:k-1},E,\beta^2)$ along with the posterior for X_k.





Proposed Solution (4/4)

procedure MLParameterEstWithTBD($\mathbf{z}_{1:k}, E_0, \beta_0^2$)
    $m \leftarrow 0,\ E_m \leftarrow E_0,\ \beta_m^2 \leftarrow \beta_0^2$
    while 0 **do**
        `Using TBD evaluate` $\nabla \log p(\mathbf{z}_{1:k} | E_m, \beta_m^2)$
        `Find` $e_m = \begin{cases} [0,1]^T, \text{if } m = 0,2,4,\ldots \\ [1,0]^T, \text{if } m = 1,3,4,\ldots \end{cases}$
        `Find` $d_m = \frac{\nabla J(E,\beta^2;\mathbf{z}_{1:k})^T e_m}{\|\nabla J(E,\beta^2;\mathbf{z}_{1:k})^T e_m\|}$
        (E_{m+1}, β_{m+1}^2) `= LineSearch(`$\mathbf{z}_{1:k}$`,`(E_m, β_m^2)`,`d_m`)`
        if $\left\|[E_{m+1}, \beta_{m+1}^2]^T - [E_m, \beta_m^2]^T\right\| < \delta$ **then**
            `break`
        end if
        $m \leftarrow m + 1$
    end while
end procedure

Example (1/2)

- Target with near constant velocity motion.
- Range and bearing resolutions of 10m and 1° yielding a 40 × 20 range-bearing intensity map.
- True values $E_T = 2$, $\beta_T^2 = 0.1$ (16dB SNR).
- Measurement history $\mathbf{z}_{1:k=200}$.

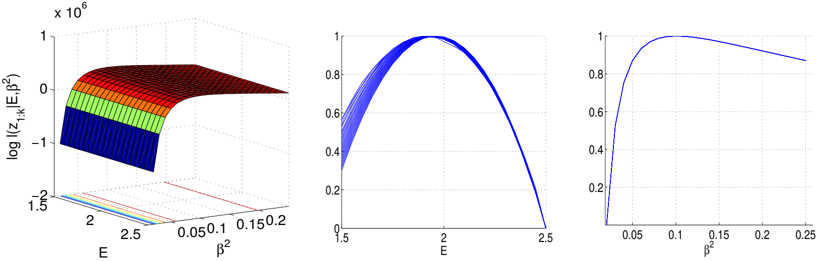


Figure: Log-likelihood surface for the example scenario evaluated over a grid with a typical measurement history.

Example (2/2)

- We used the proposed ML estimator for 200 Monte Carlo realisations.
- Convergence on the average of 3.6231 steps.
- Empirical average of the squared errors for $\hat{E}$ and $\hat{\beta}^2$ are found as 1.78×10^{-2} and 1.7×10^{-5}, respectively.

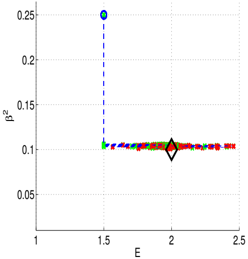


Figure: ML iterations. Initial point (blue circle), final estimates (red crosses), and intermediate results (green crosses).

Conclusions and Future Work

- We considered matched filter outputs at the receiver front-end of active sensors.
- We proposed a Maximum Likelihood scheme for jointly estimating signal amplitude and noise power.
- The gradient of the joint log-likelihood is computed using track-before-detect on the measurement history.
- Its Hessian can similarly be found leading to Newtonian maximisation iterations.
- Other possible extensions include accomodation of other Sweling target types and multiple targets, in this framework.

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