# Wideband CDMA waveforms for large MIMO sonar systems

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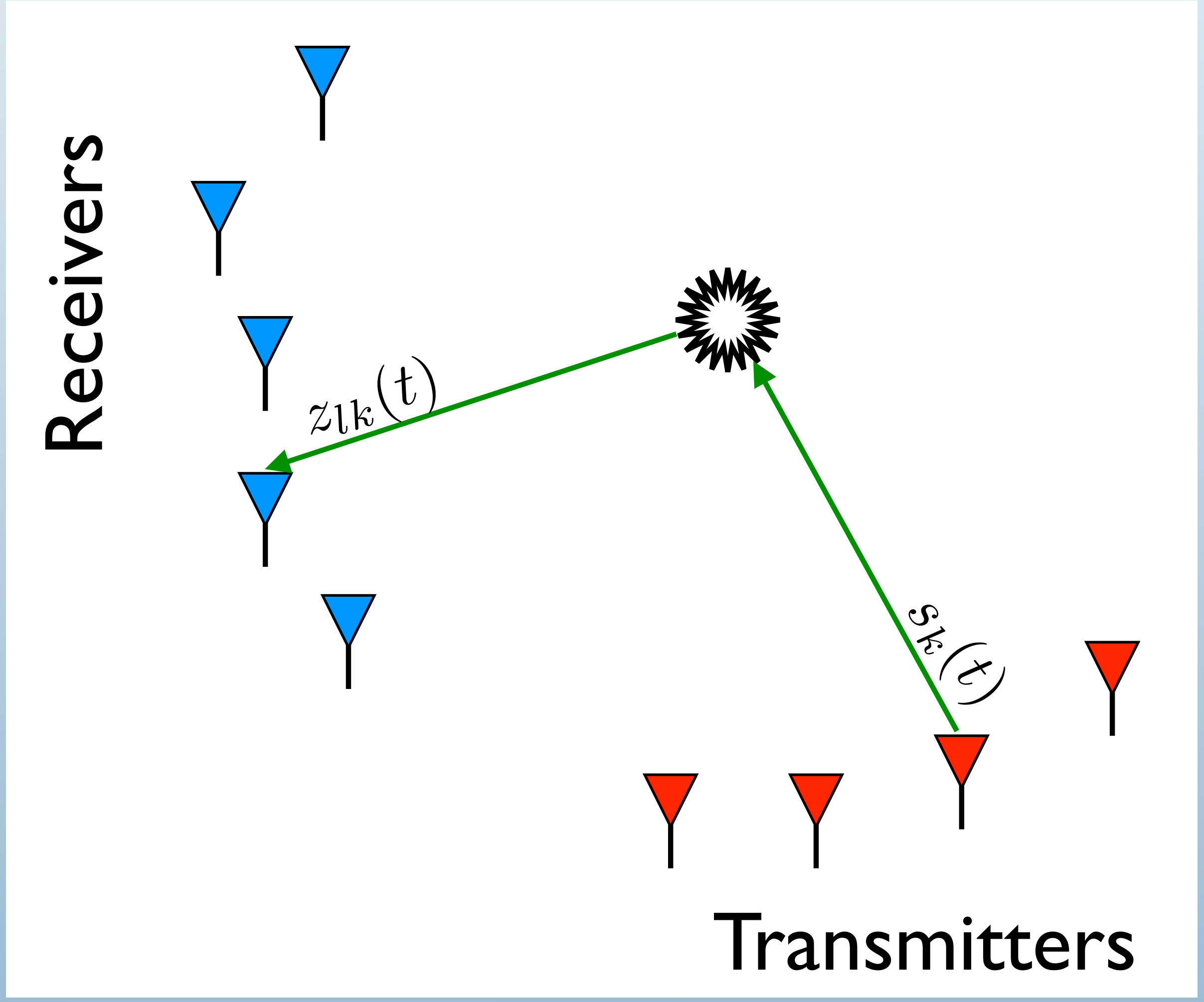
# MIMO Systems

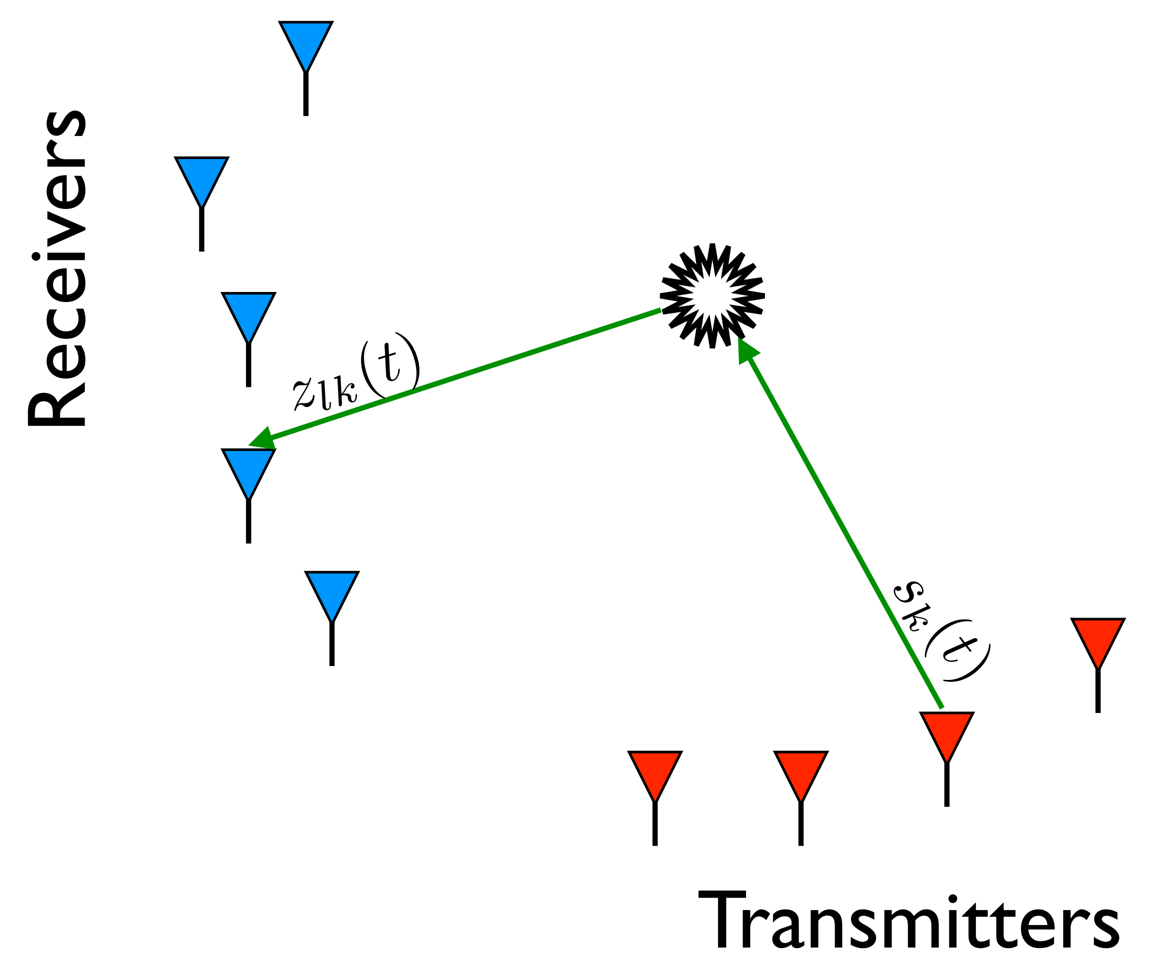
**MIMO**: **M**ultiple **I**nput **M**ultiple **O**utput

- Develop the theoretical framework for MIMO sonars
- Understand the target response from MIMO systems
- Fuse the multiple signals given by MIMO systems



# MIMO Systems

**MIMO**: **M**ultiple **I**nput **M**ultiple **O**utput



Pros:

- N x M views
- angular diversity
- bistatic views

Cons:

- complexity
- synchronisation

# Overview

- MIMO sonar model
- MIMO sonar characteristics
  - ATR
  - Super resolution
  - Independent views problem
- Orthogonal Waveforms
  - TDMA
  - FDMA
  - CDMA
- IMCS

# MIMO sonar model

We developed previously a MIMO sonar model:

$$Z_{lk}(\omega) = H_{lk}(X_0, \omega) F_\infty(\omega, \theta_l, \phi_k) S_k(\omega)$$

$Z_{lk}$ is the response of the target at the receiver $l$ from the transmitter $k$.
$H_{lk}$ is the propagation function. $X_0$ is the target centre of gravity.
$F_\infty$ is the target form function. $\theta_l$ and $\phi_k$ are the target view angles from respectively receiver $l$ and transmitter $k$.
$S_k$ is the pulse send by transmitter $k$.

The full response is given by:

$$R_l(\omega) = \sum_{k=1}^{K} Z_{lk}(\omega)$$

# MIMO sonar model

The target response from the MIMO pair *(l,k)* is then given by:

$$\begin{aligned} X_{lk}(\omega) &= R_l(\omega) S_k^*(\omega) \\ &= H_{lk}(X_0, \omega) F_\infty(\omega, \theta_l, \phi_k) \end{aligned}$$

Here we assume the orthogonality of waveforms:

$$\boxed{S_m(\omega) S_k^*(\omega) = \mathbb{1}_{m,k}(\omega)}$$

In the time domain and assuming cloud point target model, we have:

$$\sqrt{\left| \sum_{q=1}^{Q} h_{lk}^{(q)} \right|^2}$$

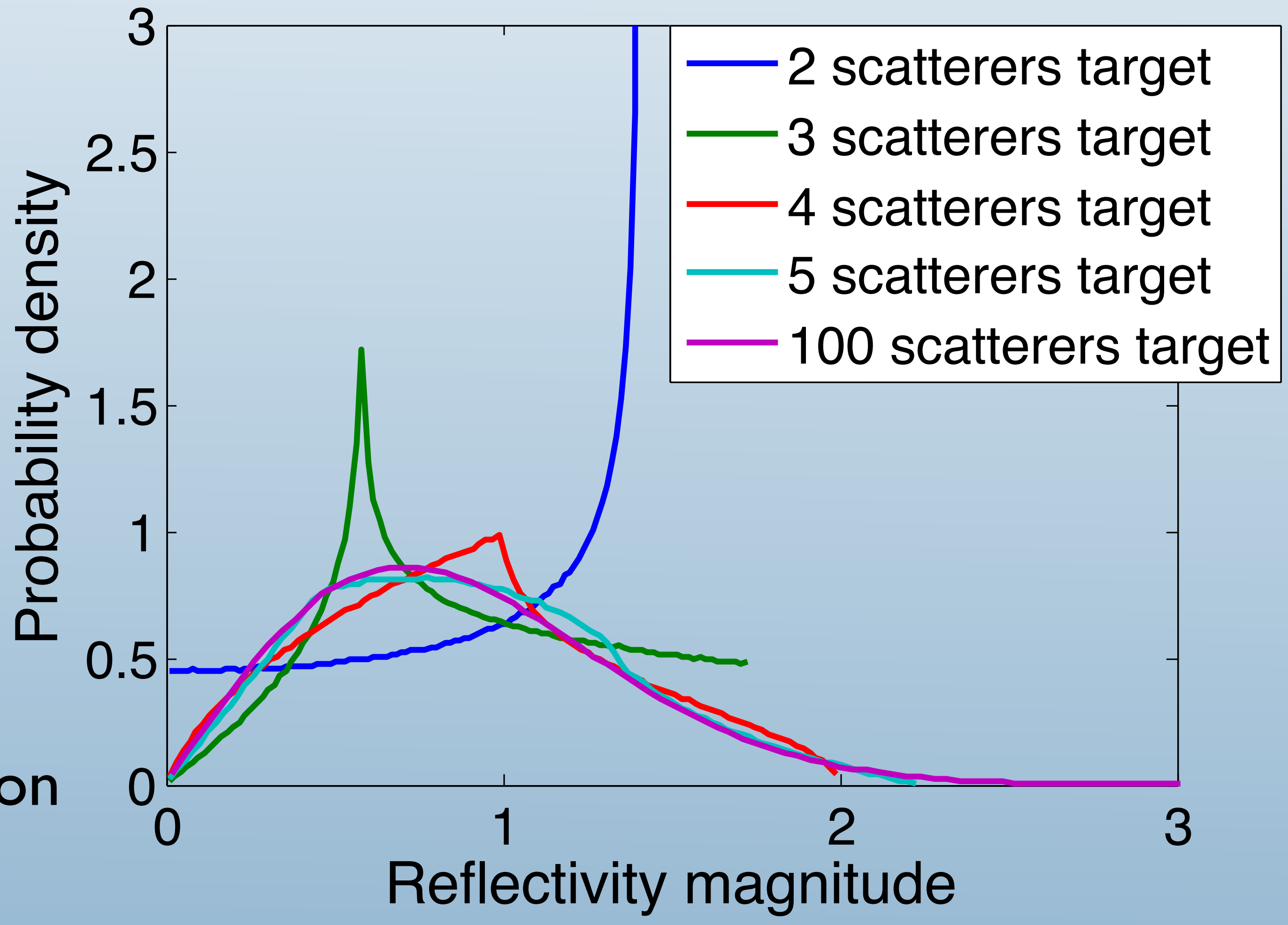
# Target recognition with MIMO systems

Study of the convergence speed of

$$\sqrt{\left|\sum_{q=1}^{Q} h_{lk}^{(q)}\right|^2}$$

Low number scatterer targets have distinguishable PDF.

With only 5 scatterers the reflectivity magnitude of the target presents a distribution very close to the Rayleigh distribution.



# Target recognition with MIMO systems

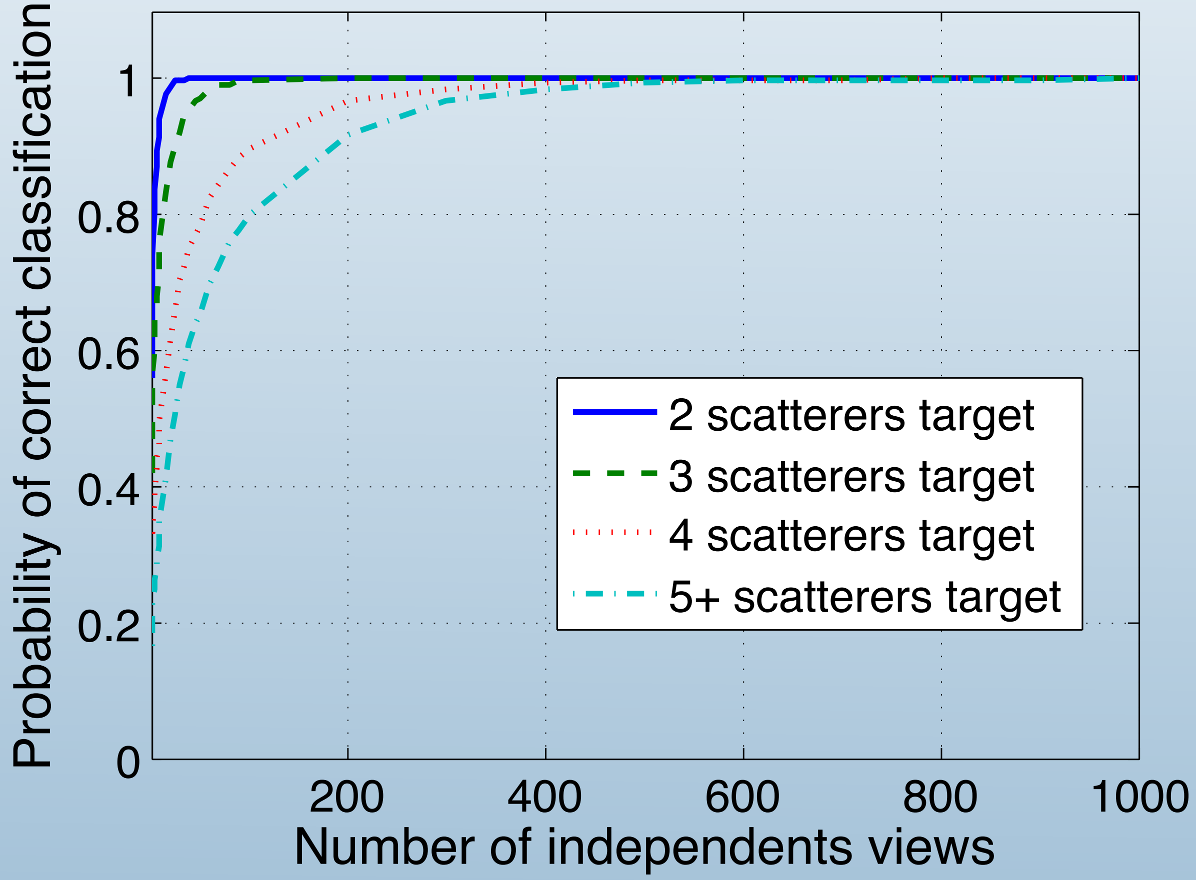
Assuming independent views we can write:

$$\mathrm{P}(X|Q_n) = \prod_{i=1}^{p} \mathrm{P}(x_i|Q_n)$$

and then we can derive from Bayes rules:

$$\boxed{\mathrm{P}(Q_n|X) = \frac{\prod_{i=1}^{p} \mathrm{P}(x_i|Q_n)}{\sum_{n=1}^{M} \mathrm{P}(X|Q_n)}}$$

| number of views | correct classification |
|---|---|
| 10 | 64% |
| 50 | 86% |
| 100 | 92% |
| 200 | 97% |
| 500 | 99.81% |
| 1000 | >99.999999 % |



Correct classification probability against the number of independent views for 4 classes of targets (2, 3, 4 and 5+ scattering points targets).

# Speckle resolution

Computing the average target intensity, we can derive the detection rule:

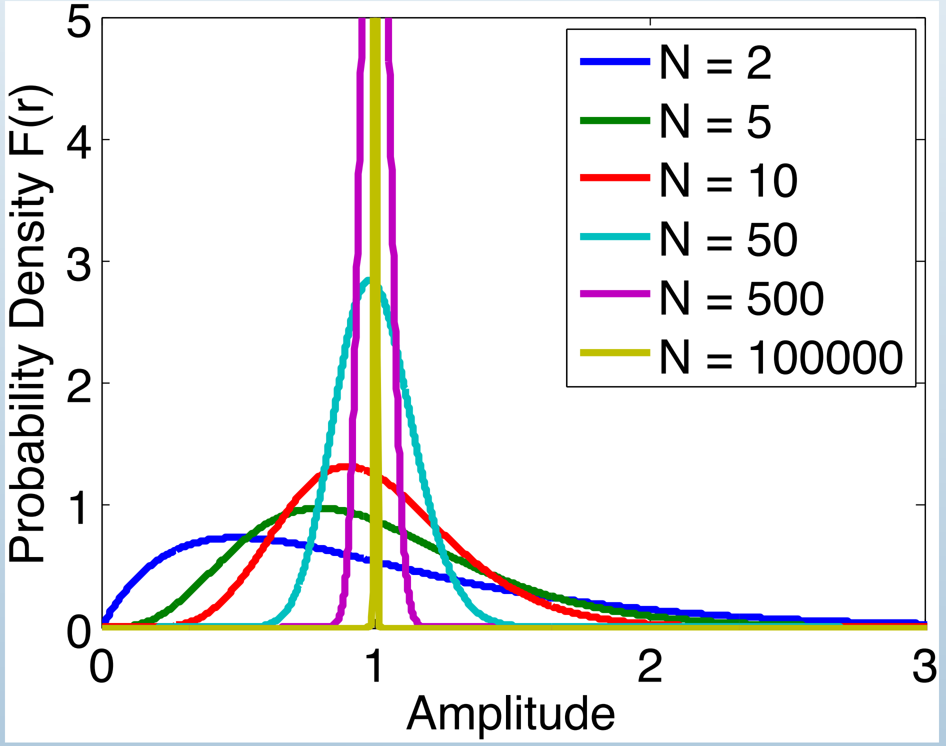
$$\mathcal{F}(\mathbf{r}) = \sum_{l,k} ||x_{lk}||^2 \sim \frac{1}{N}\sum_{n=1}^{N} \mathrm{Rayleigh}^2(\sigma)$$

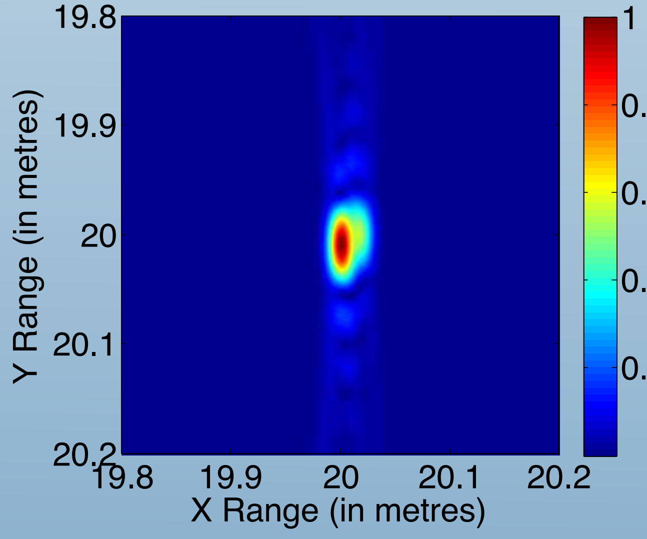
Assuming independent views and using the properties of the Rayleigh distribution, we can write:

$$\frac{1}{N}\sum_{n=1}^{N} \mathrm{Rayleigh}^2(\sigma) \sim N\Gamma(N, 2\sigma^2)$$

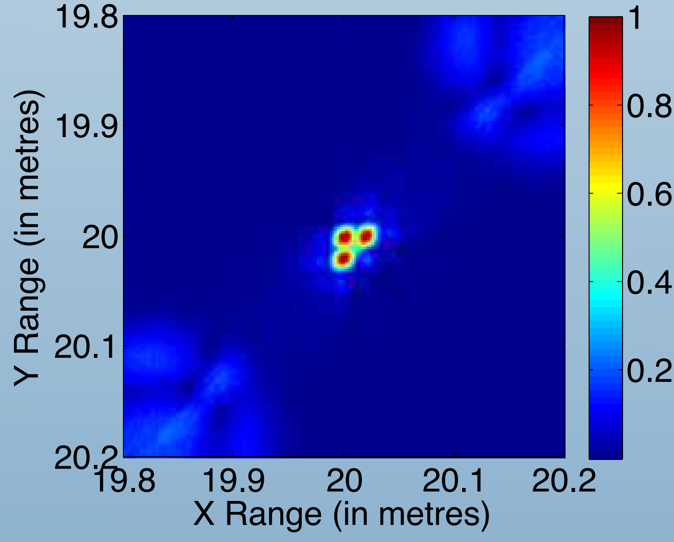
The asymptotic behaviour of the detection rule is then:

$$\lim_{N\to+\infty} \mathcal{F}(r) = \lim_{N\to+\infty} N\Gamma(N, 2\sigma^2) = \delta_1$$





SAS image



MIMO image

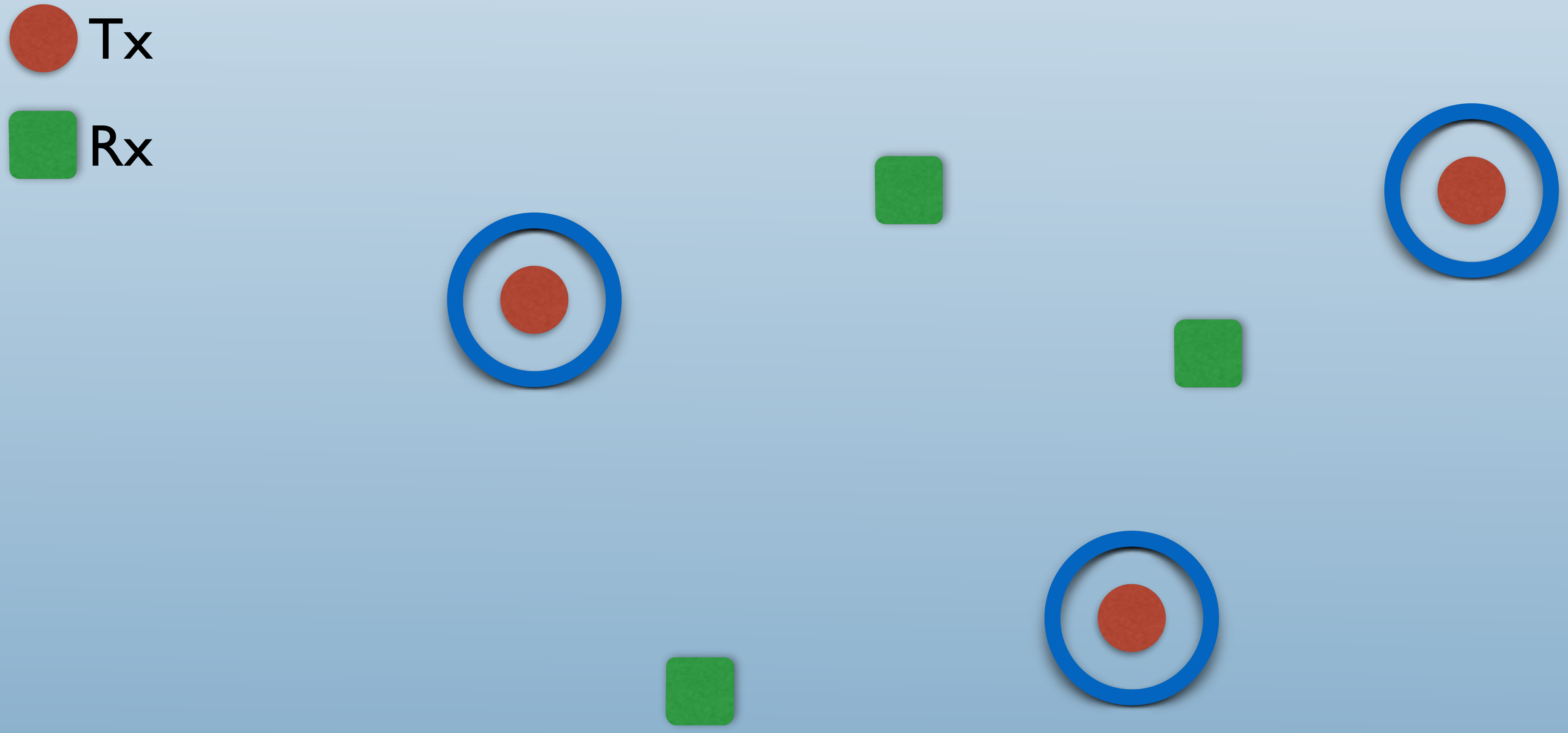
# One condition:

$$\boxed{\int_{-\infty}^{+\infty} s_i(\tau)s_j^*(t-\tau)d\tau = \delta_{i,j}}$$

# MIMO waveform strategies

# TDMA
# Time Division Multiple Access

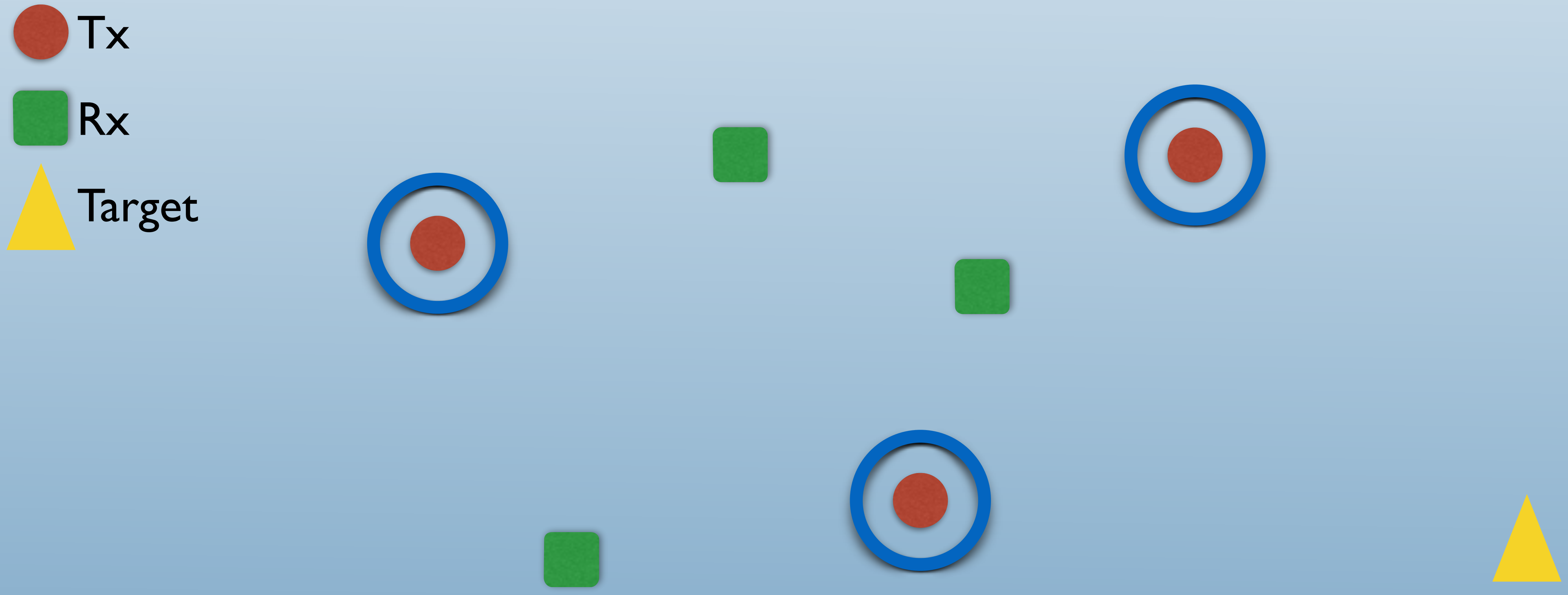
It refers to waveform sets sharing the same frequency band but not at the same time.



# TDMA
# Time Division Multiple Access

**Intrinsic problem of TDMA:**

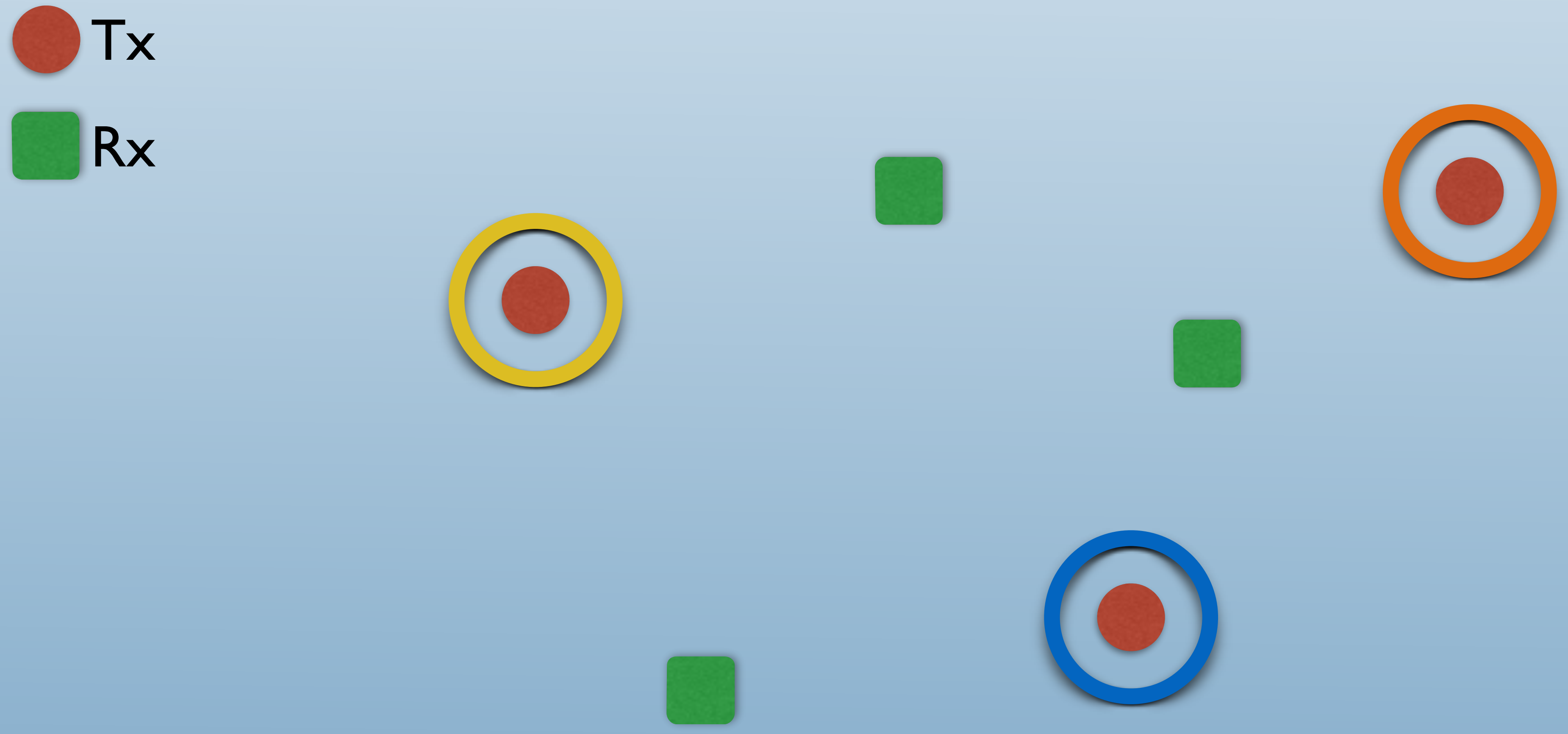
PRI (pulse repetition interval) relative the dynamic of the scene



# FDMA

## Frequency Division Multiple Access

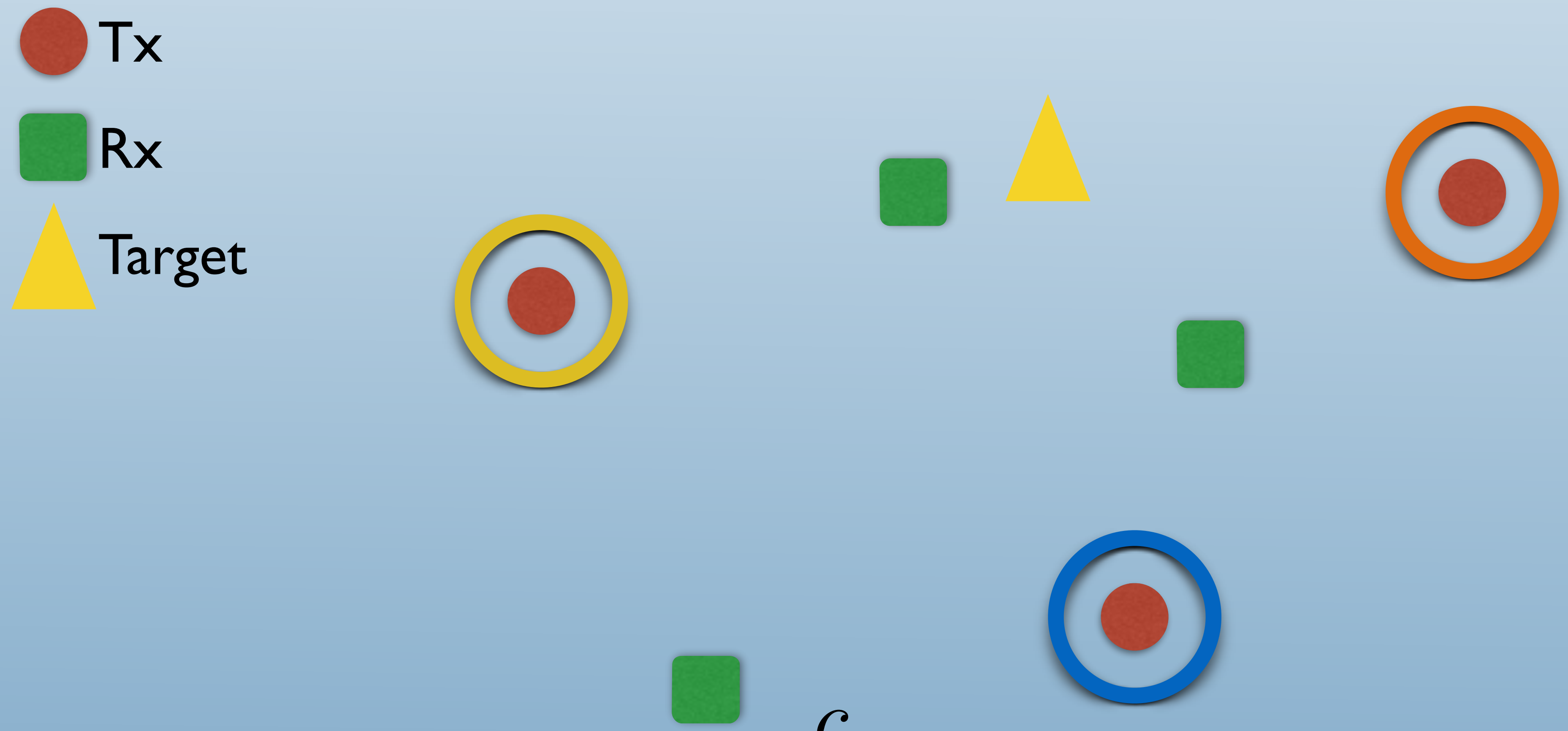
It refers to waveform sets occupying different frequency bands at the same time.



# FDMA
# Frequency Division Multiple Access

**Intrinsic problem of FDMA:**

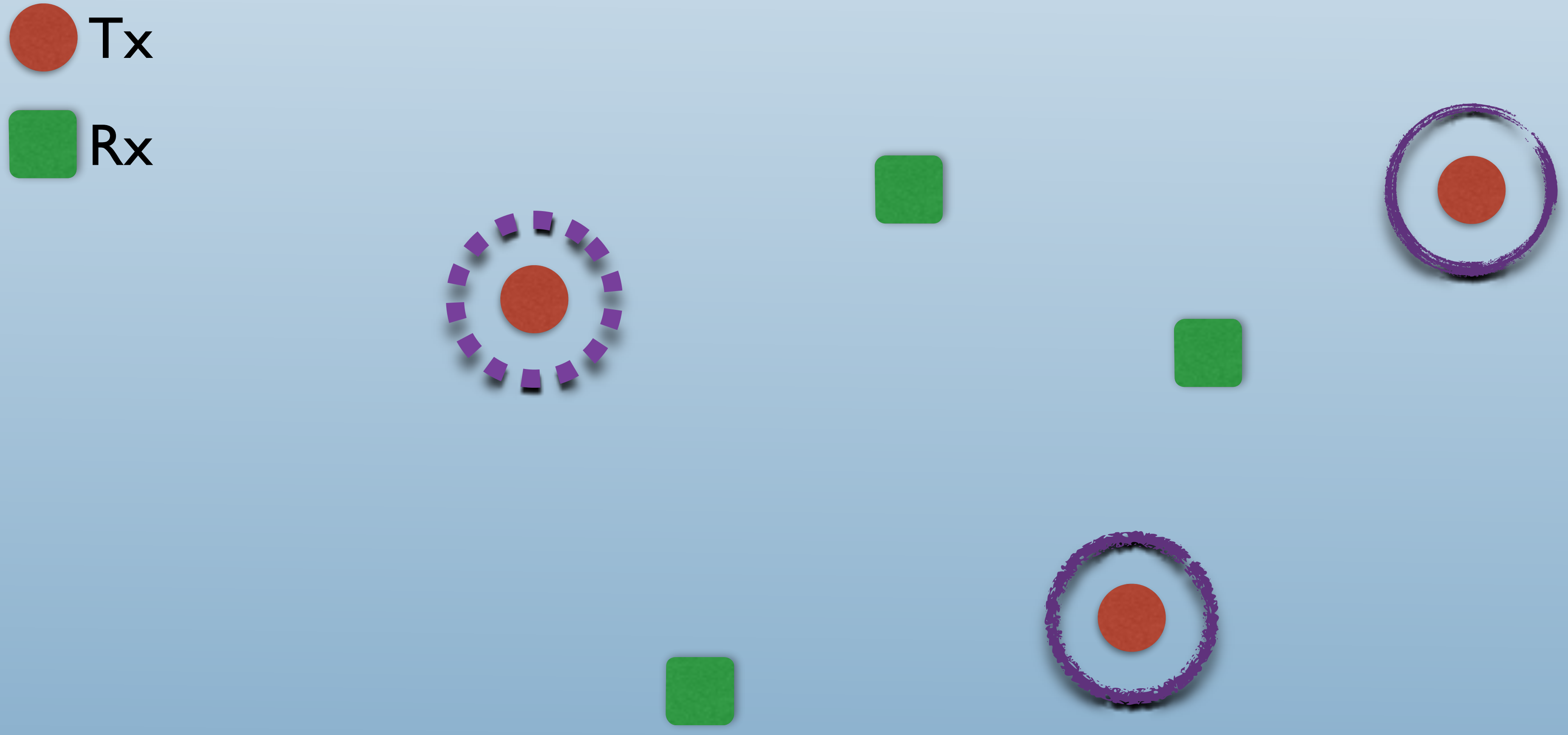
Dividing the full bandwidth by the number of transmitters.



distance resolution: $\frac{c}{2\Delta f}$ compression gain: $T\Delta f$

# CDMA
# Code Division Multiple Access

It refers to waveform sets sharing the same frequency band at the same time.



# CDMA
# Code Division Multiple Access

Diverse CDMA waveforms were proposed for radar including:

- polyphase code
- pseudo random phase code
- up/down chirps
- Baker, Gold code
- …

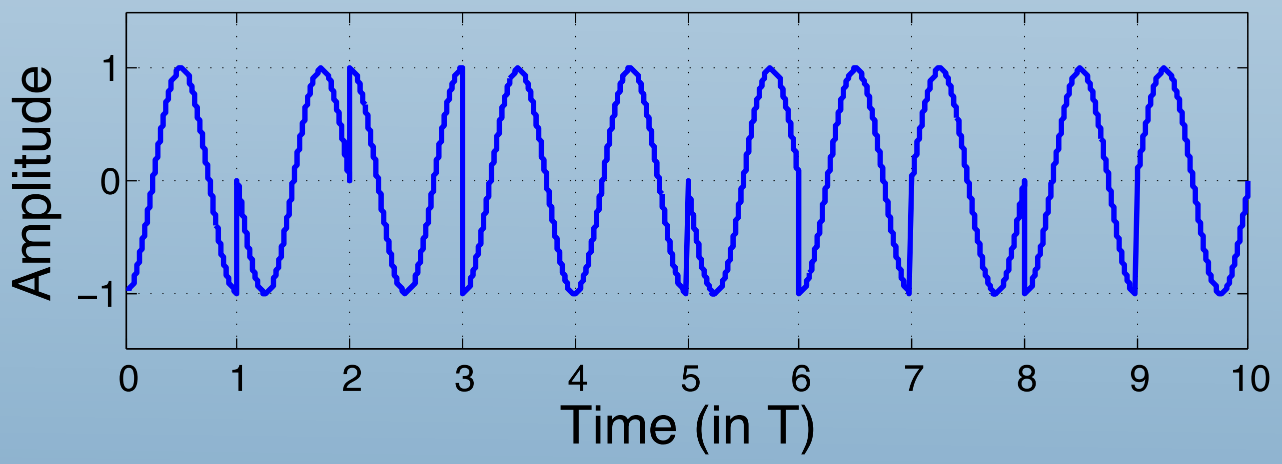
Optimisation criteria:

- sidelobe levels
- cross correlation

Constraints:

- constant amplitude

Polyphase waveform



# Interlaced Micro-Chirp Series

**CDMA requirements for wideband large MIMO sonar systems:**

1. wideband width covered by every pulses
2. “*good*” auto- and cross-correlation functions
3. possibility to generate a large number of orthogonal waveforms
4. waveforms with smooth phase transition
5. waveforms with relative constant amplitude

**Relaxed condition relative to radar:**

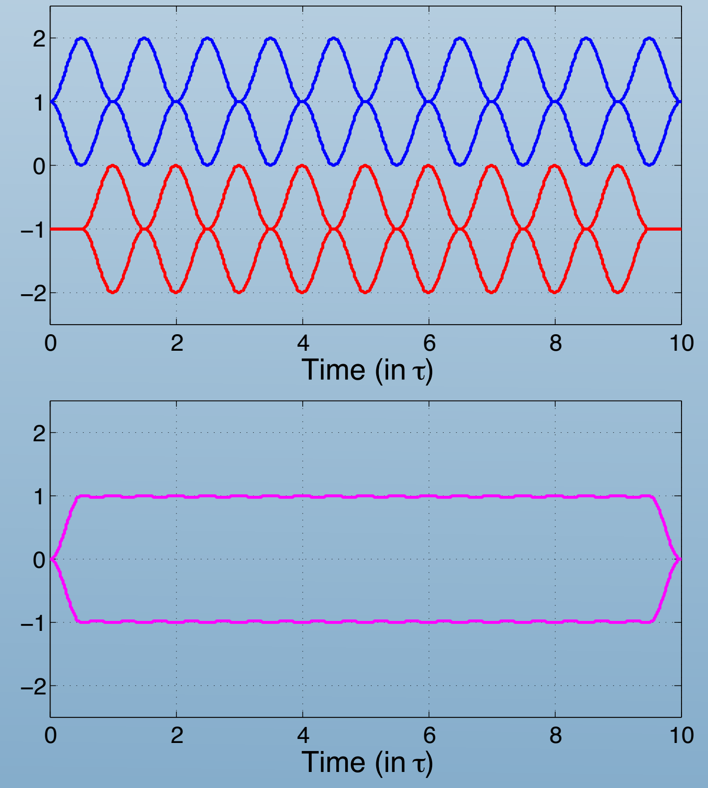
constant amplitude waveform

# Interlaced Micro-Chirp Series

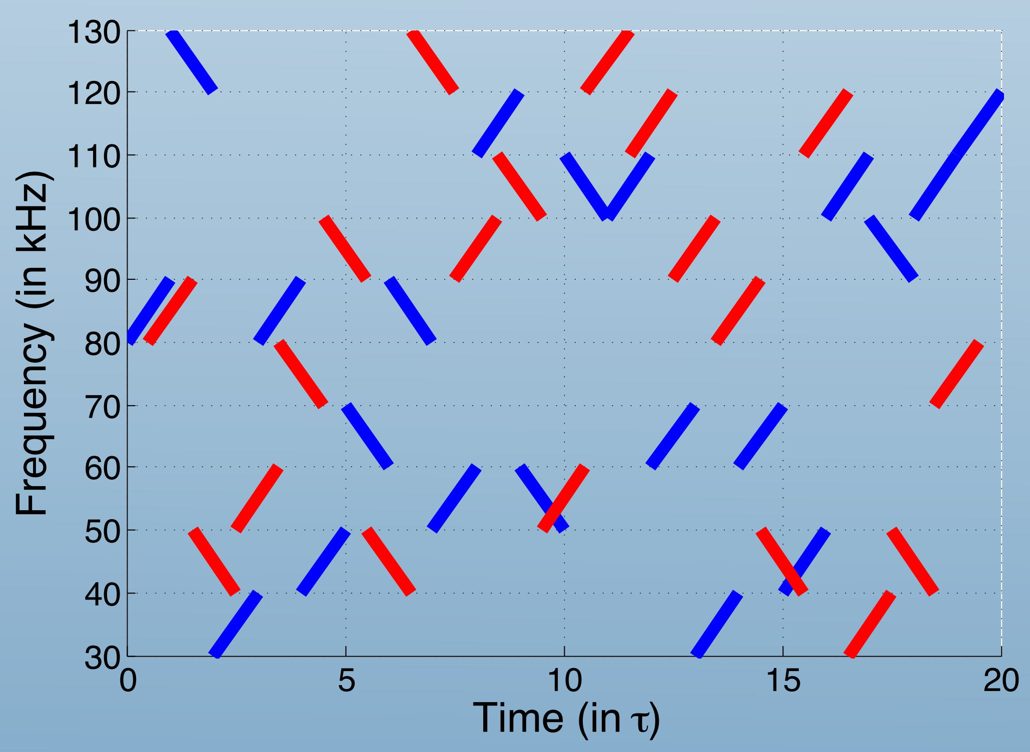
The IMCS waveform is the summation of two concatenations of micro-chirps series. Each micro-chirp has the same duration $\tau$ and the same windowing.

- smooth phase transition between each consecutive micro-chirp
- relatively constant amplitude for the overall waveform
- constrains the micro-chirp to a constant bandwidth

IMCS $\mu$-chirp structure



IMCS time-frequency structure

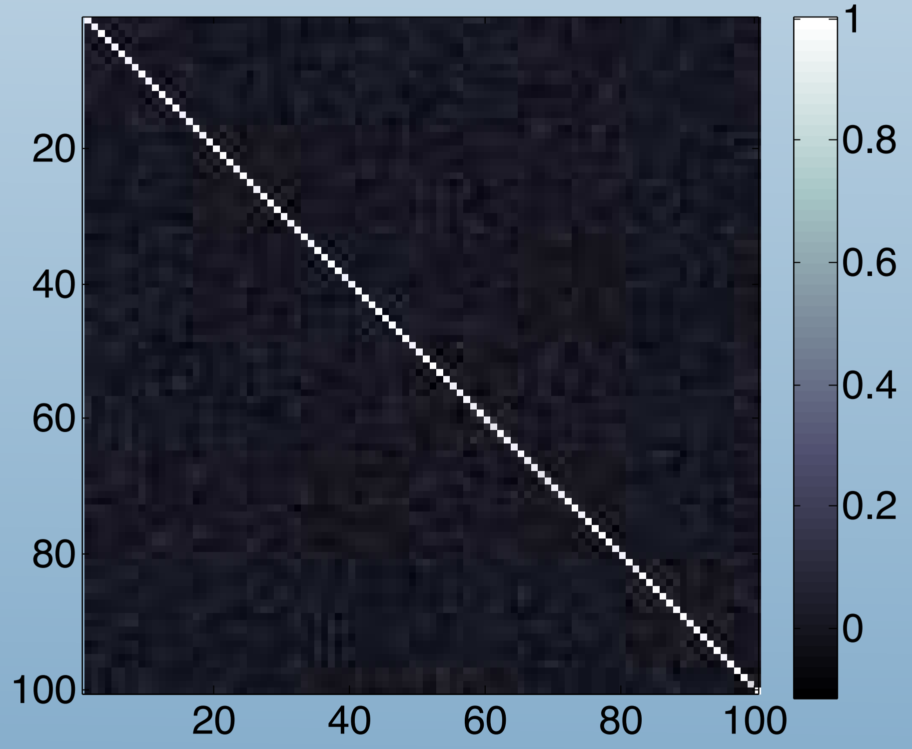


# Interlaced Micro-Chirp Series

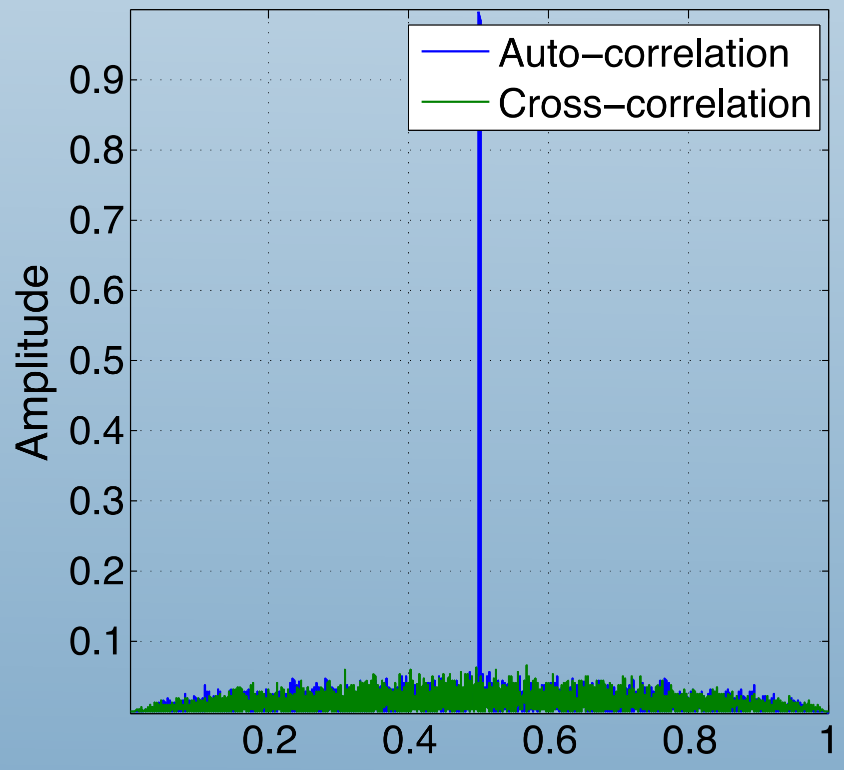
We computed 100 different waveforms for

B = [30 kHz - 130 kHz], $\tau = 10^{-4}$s, $N_B = 10$ , $N_\tau = 90$.

Covariance matrix



Auto and cross correlation



# Conclusions

- Importance of orthogonal waveforms for MIMO systems
- Orthogonal waveform strategy
- IMCS for large MIMO sonar systems