Radar Imaging With Quantized Measurements Based on Compressed Sensing

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Motivation

High-resolution radar imaging requires high-rate analog-to-digital converter (ADC) , but

- the dynamic range of ADC degrades as a function of the sampling rate, and
- the large volumes of data produced by high-rate ADC raise the burden of storing and transmission devices.

Radar Imaging With Low-bit Quantized Data

Advantage	Disadvantage
Enable high-rate ADC; Reduce the amount of data for storage and transmission.	Large quantization error; Reduce the dynamic range of radar image.

One possible solution: Quantized Compressed Sensing (QCS)

QCS Radar Imaging

Classic CS Radar Imaging

Objective: reconstruct $\boldsymbol{x}$ from $\boldsymbol{y}=\boldsymbol{A}\boldsymbol{x}+\boldsymbol{n}$

Method: l_1-regularization

$$\min \quad \frac{1}{2}\left\|\boldsymbol{A}\boldsymbol{x}-\boldsymbol{y}\right\|_2^2+\lambda\sum_{i=1}^{N}\left[x_i^2+x_{i+N}^2\right]^{1/2}$$

QCS Radar Imaging

Objective: reconstruct $\boldsymbol{x}$ from $\boldsymbol{l}\leqslant \boldsymbol{A}\boldsymbol{x}+\boldsymbol{n}\leqslant \boldsymbol{u}$

Method: maximum *a posteriori* (MAP) estimation

$$\min \; -\ln p\left(\boldsymbol{q}|\boldsymbol{x}\right)-\ln p\left(\boldsymbol{x}\right)$$

Two assumptions:

$\boldsymbol{n}_i$ are iid Gaussian random variables

$$p\left(\boldsymbol{q}|\boldsymbol{x}\right)=\prod_{i=1}^{2M}\left[\Phi\left(\frac{-\boldsymbol{a}_i^T\boldsymbol{x}+\boldsymbol{u}_i}{\sigma}\right)-\Phi\left(\frac{-\boldsymbol{a}_i^T\boldsymbol{x}+\boldsymbol{l}_i}{\sigma}\right)\right]$$

Φ is the cumulative distribution function (CDF) of the standard normal distribution.

The target reflectivity vector is sparse. The l_1-norm can be used to enforce the sparsity.

$$-\ln p\left(\boldsymbol{x}\right)\propto\left\|\boldsymbol{f}\right\|_1=\sum_{i=1}^{N}\sqrt{x_i^2+x_{i+N}^2}$$

$$\min -\sum_{i=1}^{2M}\ln\left[\Phi\left(\frac{-\boldsymbol{a}_i^T\boldsymbol{x}+\boldsymbol{u}_i}{\sigma}\right)-\Phi\left(\frac{-\boldsymbol{a}_i^T\boldsymbol{x}+\boldsymbol{l}_i}{\sigma}\right)\right]+\lambda\sum_{i=1}^{N}\sqrt{x_i^2+x_{i+N}^2}$$

a convex, unconstrained optimization problem

Basic Radar Observation Model

$$\underbrace{\underset{\text{received radar data}}{\boldsymbol{s}} = \underset{\text{measurement matrix}}{\boldsymbol{U}}\;\underset{\text{target reflectivity}}{\boldsymbol{f}} + \underset{\text{additive noise}}{\boldsymbol{e}}}_{\text{Complex-valued vectors or matrices}}$$

$\mathcal{R}\{\cdot\}$ – real part operator $\quad\mathcal{I}\{\cdot\}$ – imaginary part operator

$$\boldsymbol{y}=\begin{bmatrix}\mathcal{R}\{\boldsymbol{s}\}\\ \mathcal{I}\{\boldsymbol{s}\}\end{bmatrix},\ \boldsymbol{A}=\begin{bmatrix}\mathcal{R}\{\boldsymbol{U}\} & -\mathcal{I}\{\boldsymbol{U}\}\\ \mathcal{I}\{\boldsymbol{U}\} & \mathcal{R}\{\boldsymbol{U}\}\end{bmatrix},\ \boldsymbol{x}=\begin{bmatrix}\mathcal{R}\{\boldsymbol{f}\}\\ \mathcal{I}\{\boldsymbol{f}\}\end{bmatrix},\ \boldsymbol{n}=\begin{bmatrix}\mathcal{R}\{\boldsymbol{e}\}\\ \mathcal{I}\{\boldsymbol{e}\}\end{bmatrix}$$

$$\underbrace{\boldsymbol{y}=\boldsymbol{A}\boldsymbol{x}+\boldsymbol{n}}_{\text{Real-valued vectors or matrices}}$$

Let Q denote the quantizer function:

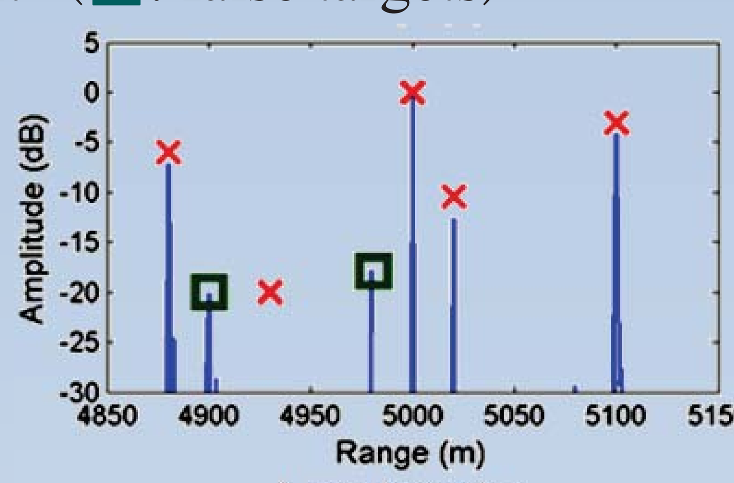
$$\boldsymbol{q}=Q\left(\boldsymbol{A}\boldsymbol{x}+\boldsymbol{n}\right)$$

Let $\boldsymbol{l}$ and $\boldsymbol{u}$ denote the lower and upper thresholds associated with $\boldsymbol{q}$, respectively.

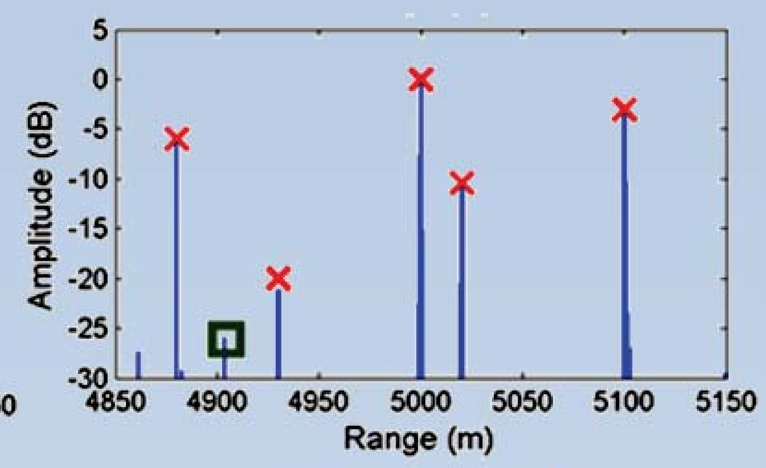
$$\boldsymbol{l}\leq\boldsymbol{A}\boldsymbol{x}+\boldsymbol{n}\leq\boldsymbol{u}$$

Simulation

1-bit（□: false targets）

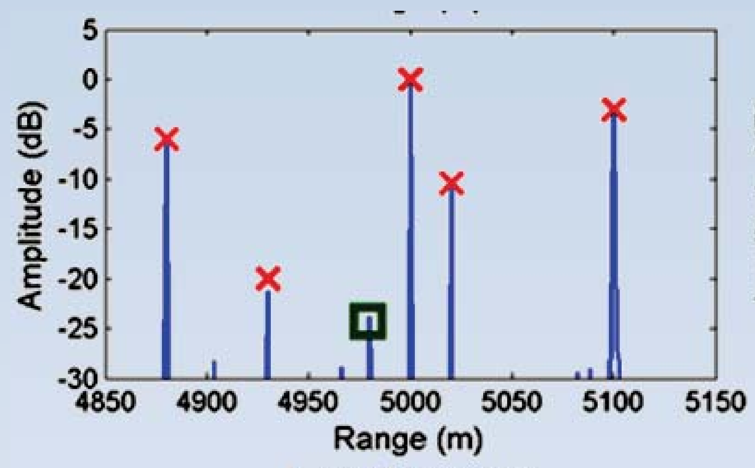


l_1-regularization

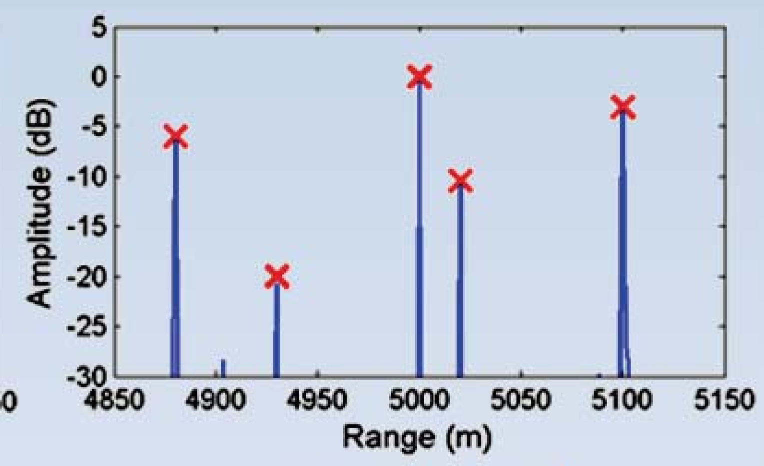


proposed approach

2-bit（□: false targets）

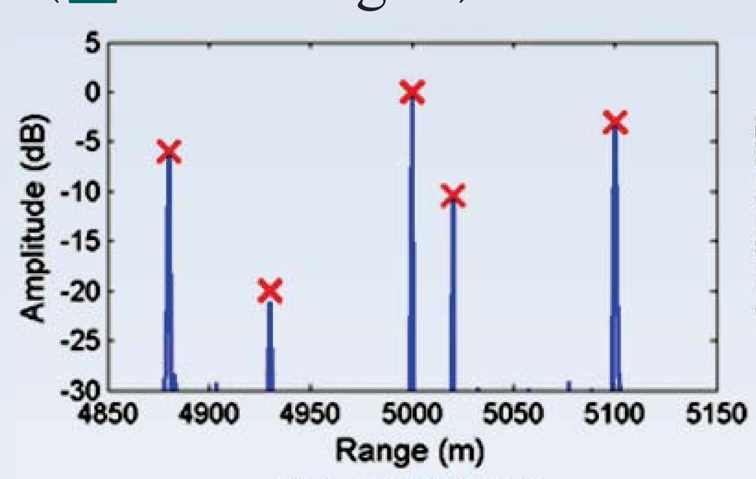


l_1-regularization

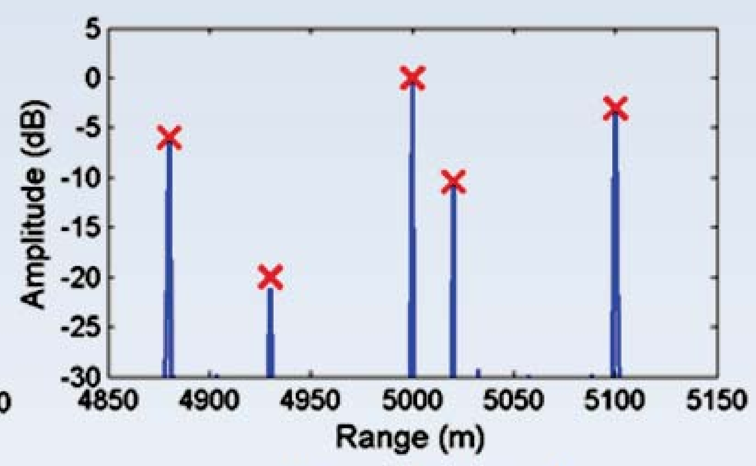


proposed approach

3-bit（□: false targets）



l_1-regularization



proposed approach

For coarsely quantized data like 1-bit and 2-bti quantization, the proposed method outperforms l_1-regularization.

For high-resolution quantization,

$$-\ln p\left(\boldsymbol{q}|\boldsymbol{x}\right)\approx\frac{1}{2}\left\|\boldsymbol{A}\boldsymbol{x}-\boldsymbol{y}\right\|_2^2+\text{Constant}$$

Thus the proposed method and l_1-regularization are approximately equivalent.