Shortening of Paraunitary Matrices Obtained by Polynomial Eigenvalue Decomposition Algorithms

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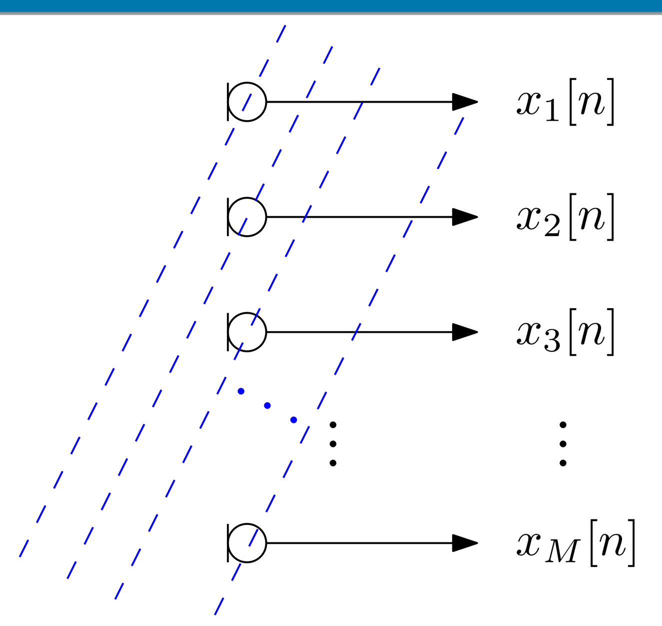
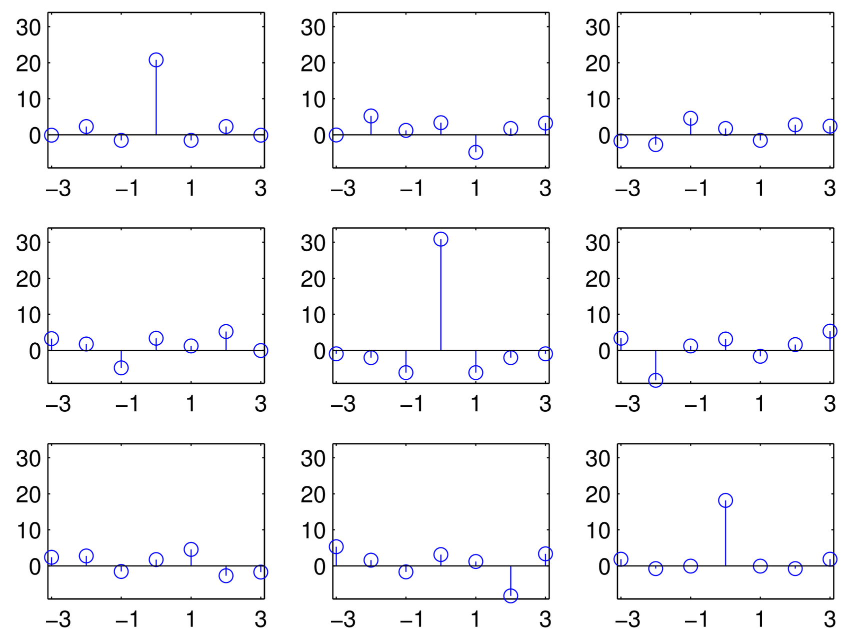
Background



- Space-time covariance matrix: $\mathbf{R}[\tau] = \mathcal{E}\{\mathbf{x}[n]\mathbf{x}^{\mathrm{H}}[n-\tau]\}$
- Matrix of auto- & cross-correlation sequences
- Symmetry $\mathbf{R}[\tau] = \mathbf{R}^{\mathrm{H}}[-\tau]$

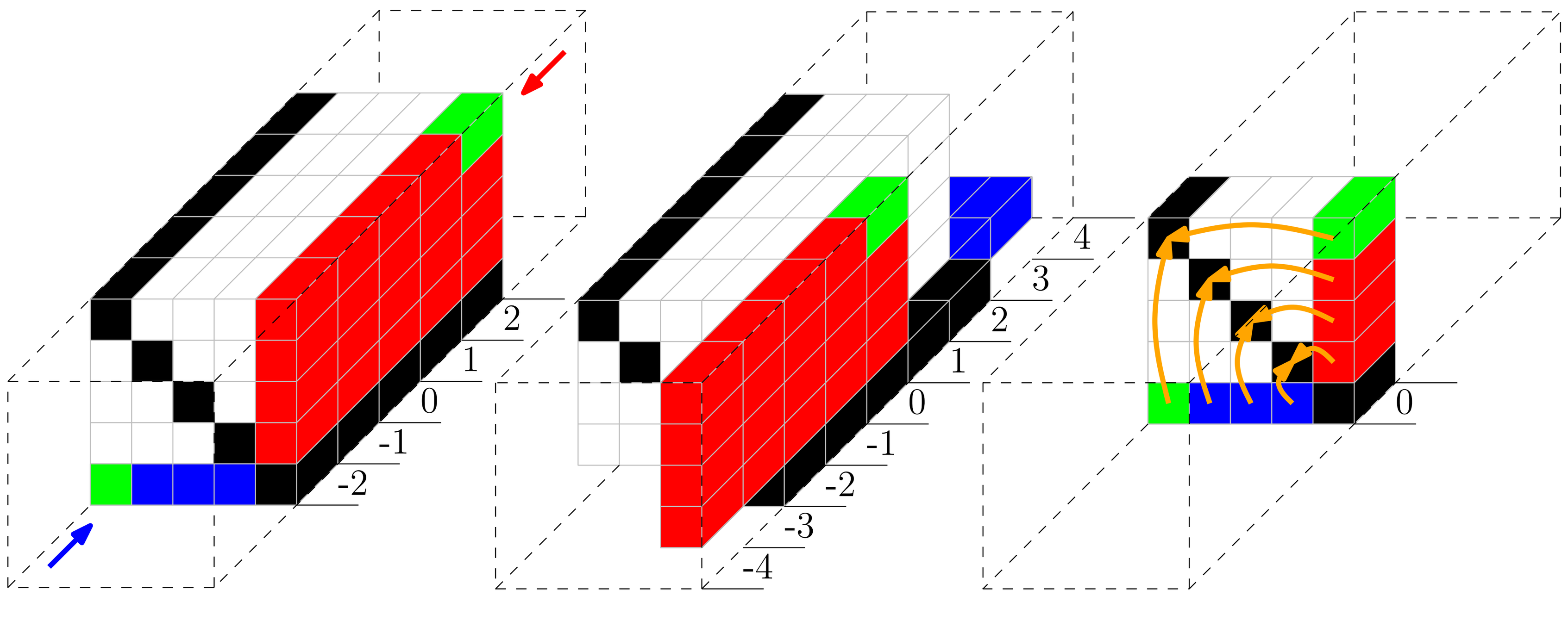


- Cross spectral density $\boldsymbol{R}(z) = \sum_{-\tau}^{\tau} \mathbf{R}[\tau]z^{-\tau}$ is a polynomial matrix.
- Parahermitian $\tilde{\boldsymbol{R}} = \boldsymbol{R}^{\mathrm{H}}(z^{-1}) = \boldsymbol{R}(z)$
- Approximate Polynomial EVD [1]: $\boldsymbol{R}(z) \approx \boldsymbol{Q}(z)\boldsymbol{D}(z)\tilde{\boldsymbol{Q}}(z)$

Iterative PEVD Algorithms

Iterative PEVD algorithms consist of three major steps:

1. Determine the elements to be shifted onto the zero lag
2. Shift the appropriate row(s) and column(s) onto the zero lag
3. Transfer energy from the zero lag onto the diagonal



1. $\{k^{(i)},\tau^{(i)}\} = \arg\max_{k,\tau} \|\hat{\mathbf{s}}_k^{(i-1)}[\tau]\|_\infty$ 2. $\mathbf{S}^{(i)\prime}(z) = \tilde{\boldsymbol{\Lambda}}^{(i)}(z)\mathbf{S}^{(i-1)}(z)\boldsymbol{\Lambda}^{(i)}(z)$ 3. $\mathbf{S}^{(i)}(z) = \mathbf{Q}^{(i)\mathrm{H}}\mathbf{S}^{(i)\prime}(z)\mathbf{Q}^{(i)}$

- Second order sequential best rotation (SBR2) [1] step 3 done using a Jacobi transformation applied to all lags.
- The sequential matrix diagonalisation (SMD) [2] algorithm uses a full EVD of the zero lag (applied to all lags) for step 3.

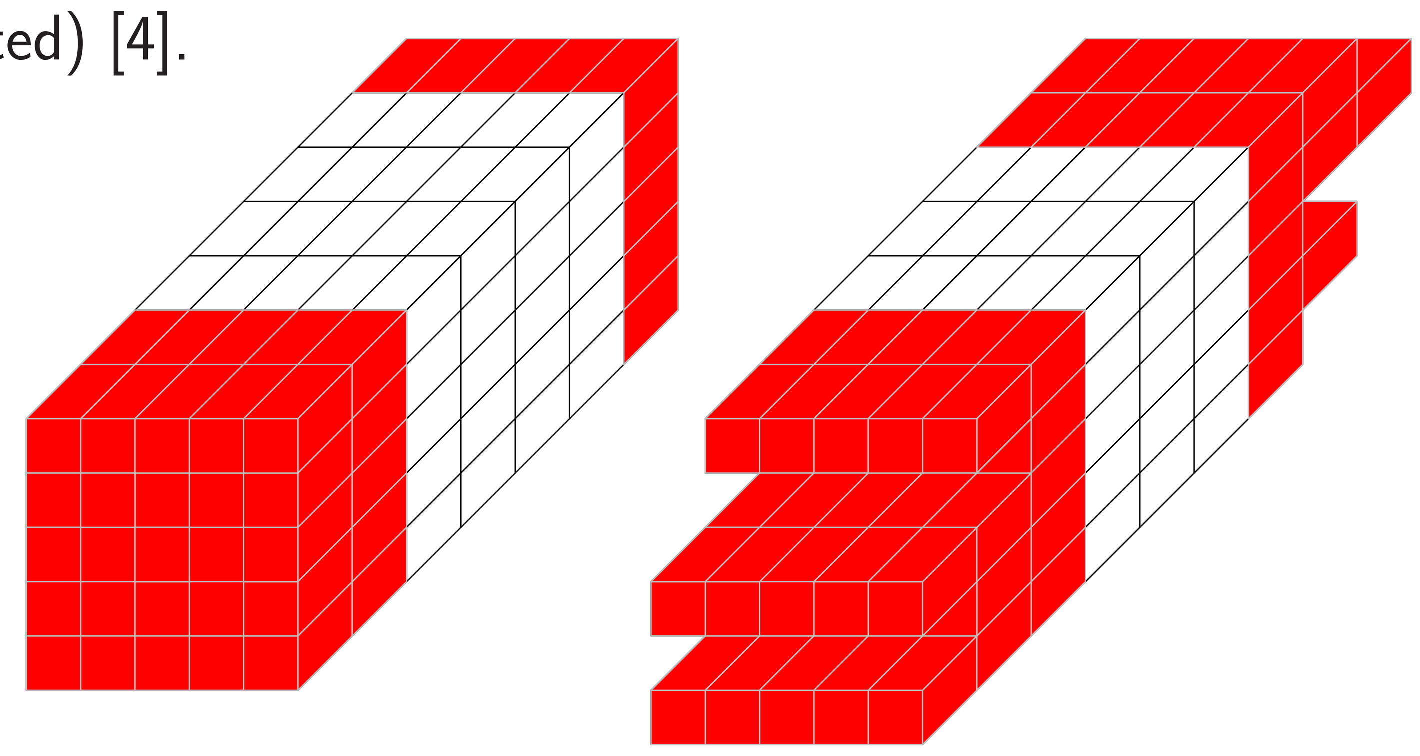
The product of these steps over I iterations provides the paraunitary matrix:

$$\boldsymbol{Q}(z) = \prod_{i=1}^{I} \boldsymbol{\Lambda}^{(i)}(z)\mathbf{Q}^{(i)} \quad . \qquad (1)$$

Truncation Methods

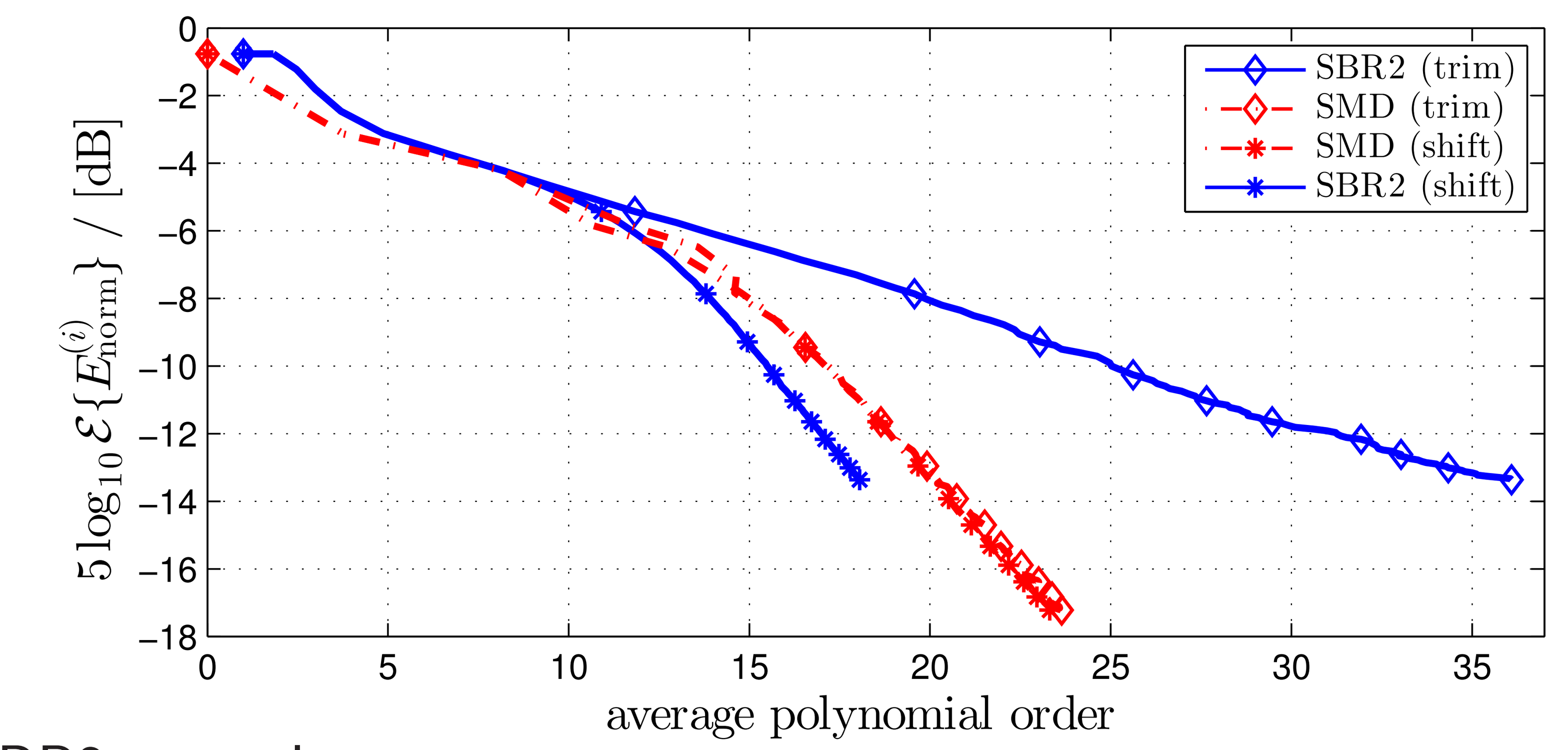
Two truncation approaches:

- Lag based trim (all rows truncated by the same amount) [3].
- Our new row-shift truncation (rows truncated individually then shifted) [4].

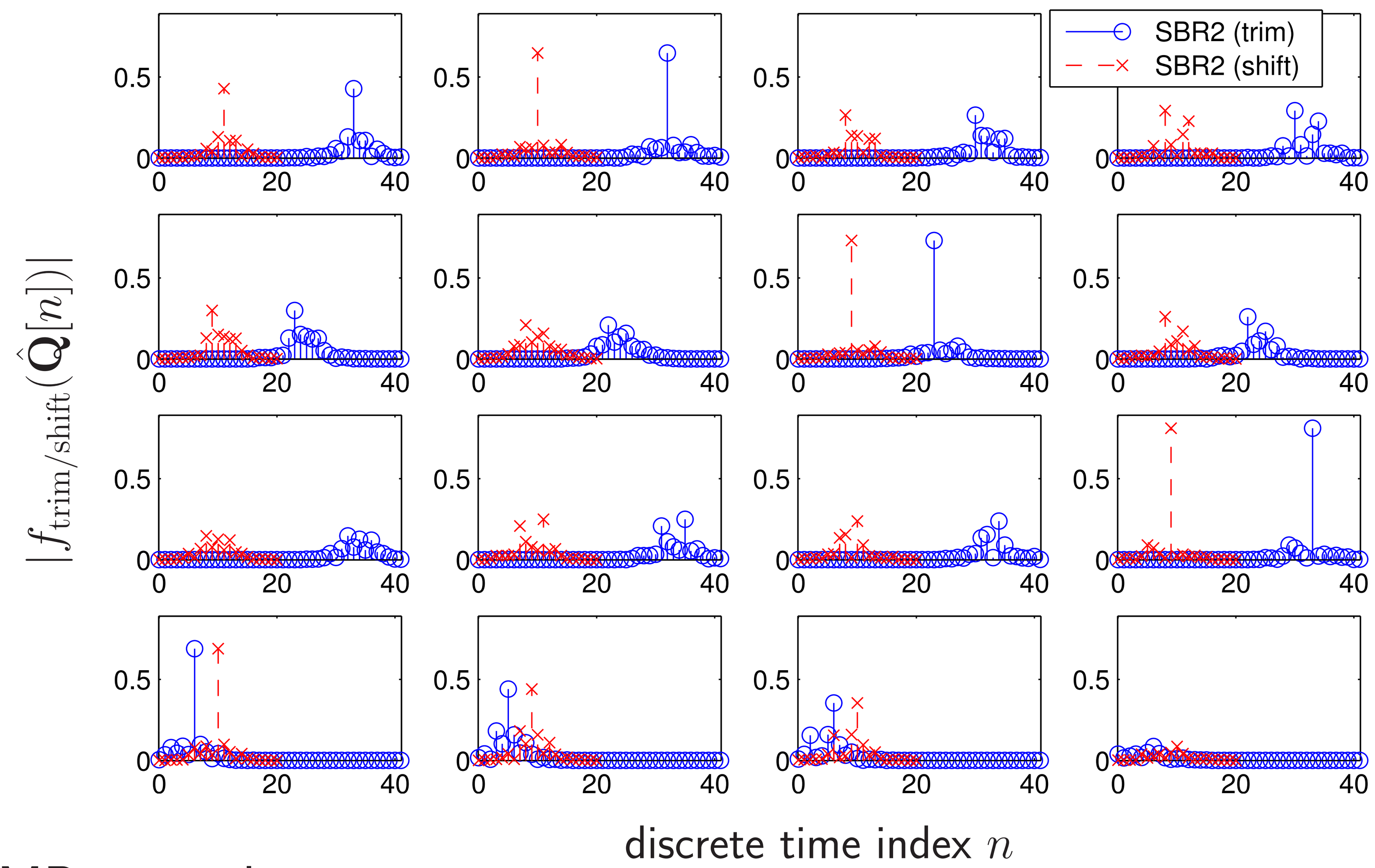


Results

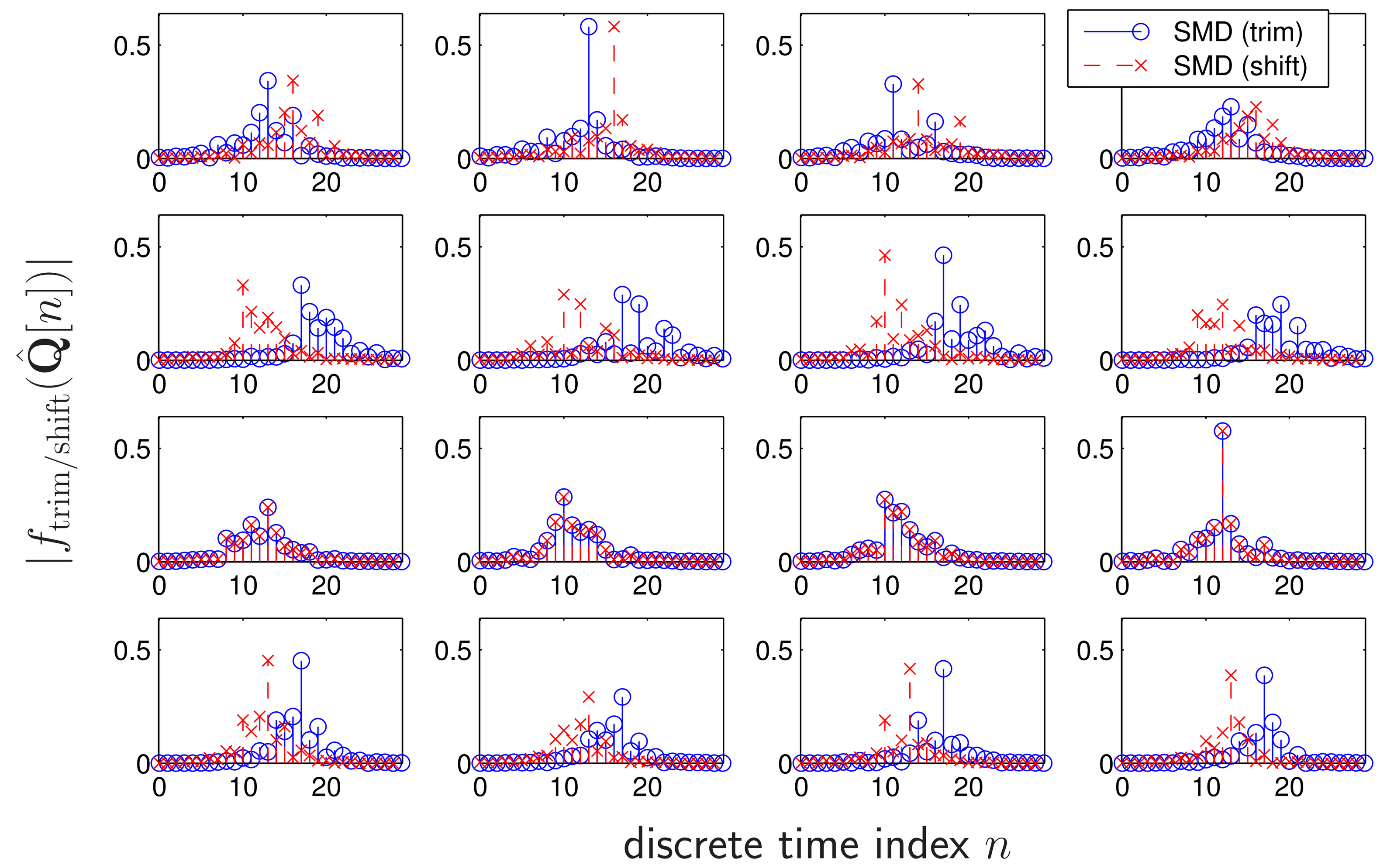
Off-diagonal energy vs. paraunitary matrix order:



SBR2 example:



SMD example:



Conclusions

Applying row-shift truncation to paraunitary matrices generated by SBR2 and SMD give contrasting results. The order of paraunitary matrices produced by SBR2 are significantly reduced by the new truncation method whereas only a marginal benefit is obtained from the SMD equivalent. Previously SMD has been favoured for low order paraunitary matrices; here the SBR2 algorithm with row-shift truncation can generate lower order paraunitary matrices.

References

[1] J. G. McWhirter et al. An EVD Algorithm for Para-Hermitian Polynomial Matrices. *IEEE Trans. on Signal Processing*, May 2007.
[2] S. Redif et al. Sequential Matrix Diagonalisation Algorithms for Polynomial EVD of Parahermitian Matrices. *IEEE Trans. on Signal Processing*, Jan 2015.
[3] C. H. Ta et al. Shortening the Order of Paraunitary Matrices in SBR2 Algorithm. *ICICSP*, Singapore, Dec 2007.
[4] J. Corr et al. Row-Shift Corrected Truncation of Paraunitary Matrices for PEVD Algorithms. In *EUSIPCO*, Nice, France, Sept 2015.

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